EISCAT
TECHNICAL
NOTES

EISCAT UHF GEOMETRY

by

J. Murdin
Kiruna Geophysical Institute
S-981 01 Kiruna, Sweden

KIRUNA
Sweden

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Kiruna Geophysical Institute
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Introduction

The geometry of the EISCAT UHF system is described together with computer programmes used to evaluate required parameters. We also include a programme which selects the optimum linear polarisation for the system.

2. Coordinate systems

Points on or above the earth's surface are described by geographic latitude and longitude and height above sea level or by coordinates in the geocentric reference frame, fig 1. It is necessary to be able to transform from one system to the other.

Sea level is well represented by an oblate ellipsoid. The ellipse of revolution has oblateness f and semi-major (equatorial) axis a, where $f = 298.25^{-1}$ and a = 6378.16 km, Allen (1973).

The ellipse of revolution is sketched in fig 1 and described by the equation

$$x^2 + gz^2 = a^2 \qquad \text{where } g = (1-f)^{-2}$$

so $x + gz\frac{dz}{dx} = 0$

and $\frac{dz}{dx} = -\frac{x}{gz}$

so $\tan\theta = \frac{gz}{x}$ or $z = \frac{x}{g}\tan\theta$

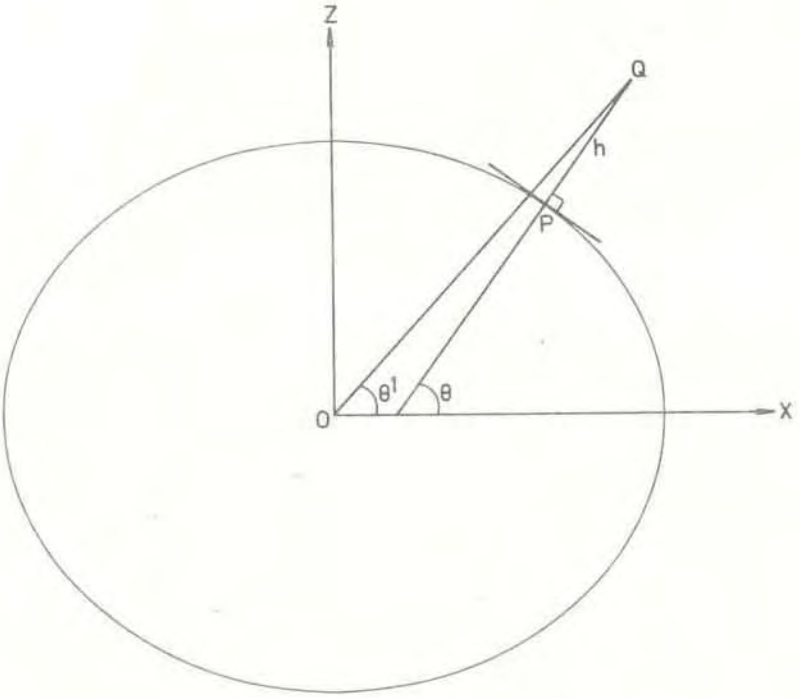

Fig 1. Relationship between geographic latitude θ and geocentric latitude θ^1.

Substituting this expression for z into the ellipse equation we can solve for x and z, thus OQ is described by

$$OQ_x = a(1 + g^{-1}\tan^2\theta)^{-1/2} + h\cos\theta$$

$$OQ_z = a(g + g^2\cot^2\theta)^{-1/2} + h\sin\theta$$

This expresses OQ in the geocentric reference frame in terms of the geographic coordinates of Q. The inverse relationship is given in a series expansion in f by Long (1974) to an accuracy of 1 part in 10^7 as

$$\theta = \theta^1 + \frac{\sin 2\theta^1}{\rho} f + \left(\frac{1}{\rho^2} - \frac{1}{4\rho}\right)\sin 4\theta^1 \; f^2$$

and

$$\frac{h}{a} = (\rho - 1) + \left(\frac{1 - \cos 2\theta^1}{2}\right) f + \left(\frac{1}{4\rho} - \frac{1}{16}\right)(1 - \cos 4\theta^1) \; f^2$$

where $\rho = \frac{OQ}{a}$

Introducing three dimensions and longitude ϕ east of Greenwich we obtain, fig 2,

$$OQ_x = (a(1 + g^{-1}\tan^2\theta)^{-1/2} + h\cos\theta)\cos\phi$$

$$OQ_y = (a(1 + g^{-1}\tan^2\theta)^{-1/2} + h\cos\theta)\sin\phi$$

$$OQ_z = a(g + g^2\cot^2\theta)^{-1/2} + h\sin\theta$$

It is also necessary to describe vectors, with origin at Q, in both the geocentric reference frame and in the local reference frame at Q. We choose the local frame as

$\hat{x}^1$ south
$\hat{y}^1$ east
$\hat{z}^1$ vertically up, see fig 2

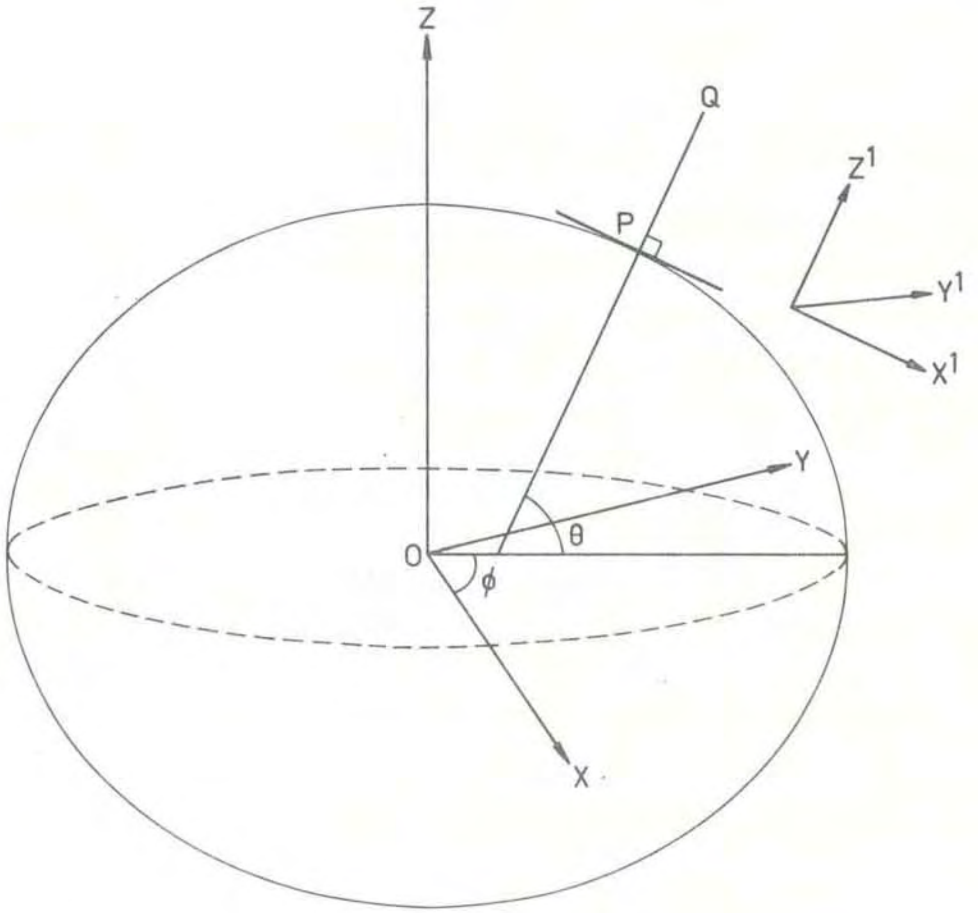

Fig 2. Geocentric and local reference frames.

The matrices for geocentric to local, R_{gl}, and local to geocentric, R_{lg}, transformations are given by

$$R_{gl} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & -\cos\theta \\ -\sin\phi & \cos\phi & 0 \\ \cos\theta\cos\phi & \cos\ \sin\phi & \sin\theta \end{bmatrix}$$

$$R_{lg} = \begin{bmatrix} \sin\theta\cos\phi & -\sin\phi & \cos\theta\cos\phi \\ \sin\theta\sin\phi & \cos\phi & \cos\theta\sin\phi \\ -\cos\theta & 0 & \sin\theta \end{bmatrix}$$

where θ and φ are the geographic latitude and longitude of Q.

3. Antenna pointing directions

The coordinates of the three antennas are listed below, Westerlund (1977).

	T Tromso	K Kiruna	S Sodankyla
latitude degrees	69.583	67.863	67.367
longitude degrees	19.21	20.44	26.65
height km	0.030	0.412	0.180

which gives in geocentric coordinates

	x	y	z	(km)
OT	2107.345	734.268	5955.091	
OK	2258.752	841.818	5885.754	
OS	2200.075	1104.118	5864.472	

and geocentric to local transformation matrices

$$R_T \begin{bmatrix} 0.884995 & 0.3083361 & -0.348850 \\ -0.329031 & 0.944319 & 0.0 \\ 0.329426 & 0.114783 & 0.937179 \end{bmatrix}$$

$$R_K \begin{bmatrix} 0.867965 & 0.323483 & -0.376823 \\ -0.349226 & 0.937038 & 0.0 \\ 0.353097 & 0.131596 & 0.926285 \end{bmatrix}$$

$$R_S \begin{bmatrix} 0.824933 & 0.413997 & -0.384827 \\ -0.448539 & 0.893763 & 0.0 \\ 0.343944 & 0.172610 & 0.927989 \end{bmatrix}$$

As an example consider the transmitting antenna at Tromso. For a given scattering point Q the vector $\overline{OQ}$ in geocentric coordinates is as defined in the previous section

Thus $\overline{TQ} = \overline{OQ} - \overline{OT}$

and in the local frame at Tromso

$$\overline{TQT} = \overline{R_T} \cdot \overline{TQ} .$$

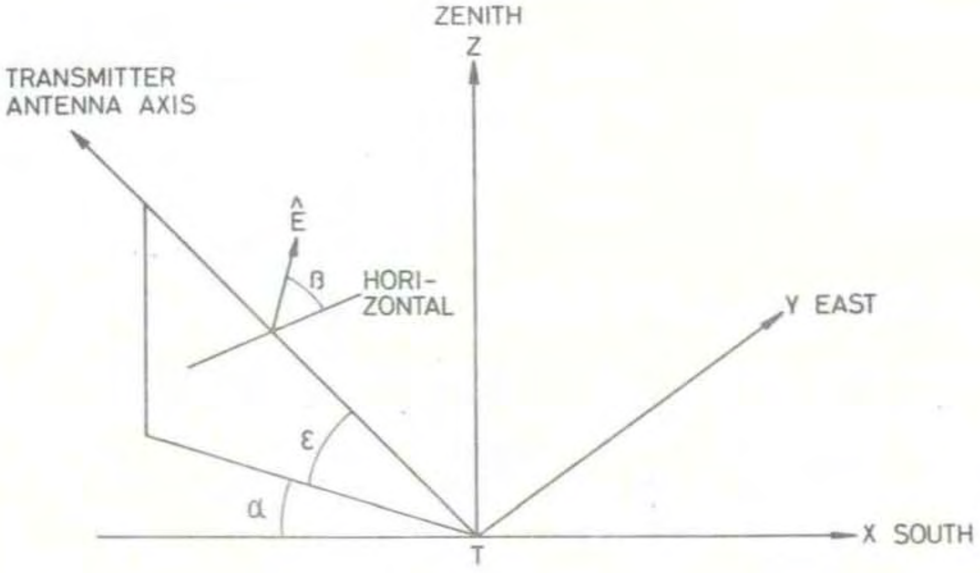

Fig 3. Antenna pointing in the local frame and polarisation of transmitter/receiver.

The antenna pointing is at elevation ε and azimuth α, see fig 3, then

$$r\cos\varepsilon\cos\alpha = -\,TQT_x$$

$$r\cos\varepsilon\sin\alpha = TQT_y$$

$$r\sin\varepsilon = TQT_z$$

which determines α and ε. r is tne range of Q from Tromso.

4. Mirror directions and plasma drift velocity

A particular receiver is sensitive to the plasma drift velocity component along the bisector of the angle between the transmitter and receiver antenna axes, the mirror direction. For the receiver at Kiruna the situation is illustrated in fig 4.

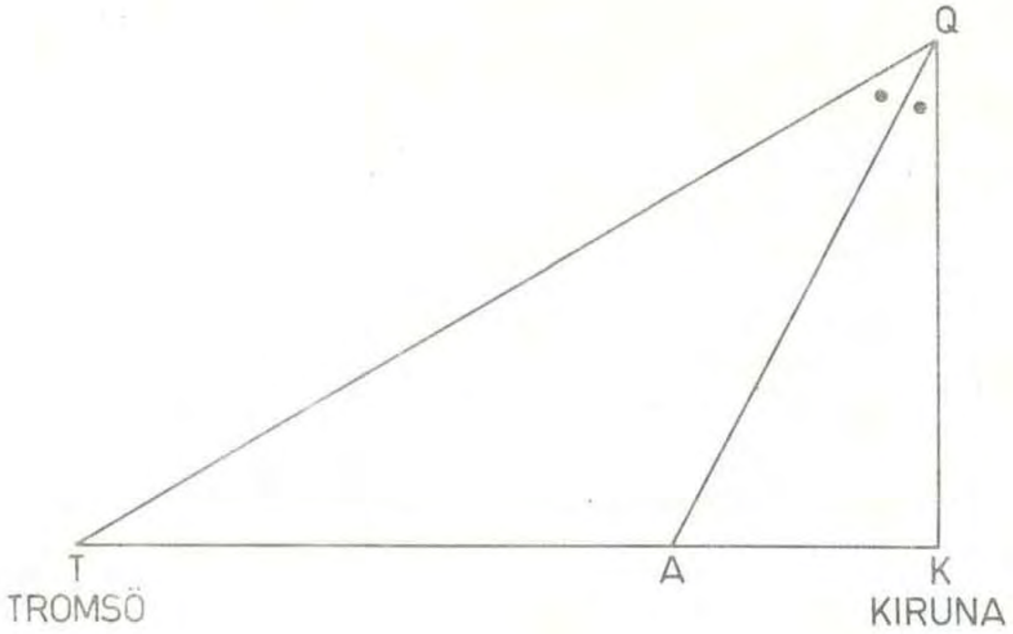

Fig 4. Mirror direction for the receiver at Kiruna.

where $\overline{AQ} = \dfrac{r_T\overline{KQ} + r_K\overline{KQ}}{r_T + r_K}$

r_T and r_K are the ranges of Q from Tromso and Kiruna.

If $\widehat{AQ}$ is this vector normalized, $\widehat{BQ}$ is the mirror direction for Sodankyla and $\widehat{TQ}$ (along the antenna axis) for Tromso then

$$\widehat{TQ} \cdot \overline{V} = V_T$$
$$\widehat{AQ} \cdot \overline{V} = V_K$$
$$\widehat{BQ} \cdot \overline{V} = V_S$$

where $\overline{V}$ is the plasma drift velocity and V_T, V_K and V_S are the components of $\overline{V}$ determined by Tromso, Kiruna and Sodankyla respectively.

These equations may be represented by

$$\overline{A}\ \overline{V} = \overline{V}_m$$

where $\overline{A}$ is a 3 x 3 matrix with rows $\widehat{TQ}$, $\widehat{AQ}$ and $\widehat{BQ}$ and the column vector

$$\overline{V}_m = \begin{bmatrix} V_T \\ V_K \\ V_S \end{bmatrix}$$

Inverting the equation we obtain

$$\overline{V} = \overline{A}^{-1}\ \overline{V}_m$$

To evaluate this expression for $\overline{V}$ all vectors must be described in the same reference frame, most conveniently the local frame at Q. It is then easy to transform $\overline{V}$ into a appropriate geomagnetic frame, e.g. a frame with axes perpendicular to the magnetic field and to the magnetic meridian, positive east $_{\perp}E$; perpendicular to the magnetic field and in the plane of the magnetic meridian, positive north $_{\perp}N$, and antiparallel to the magnetic field $_{\parallel}$ by the transformation

$$\begin{bmatrix} V_{\perp E} \\ V_{\perp N} \\ V_{\parallel} \end{bmatrix} = \overline{B}\ \overline{V} = \overline{B}\ \overline{A}^{-1}\ \overline{V}_m$$

where B is a 3 x 3 matrix with rows of unit vectors along the axes of the geomagnetic reference frame, Rishbeth (1978).

5. Transmitter and receiver polarisation

The power scattered in a particular direction depends on the angle ψ between the electric vector of the transmitted signal and the scattered direction as the factor $\sin^2\psi$. If the transmitted signal is linearly polarised one possible choice of polarisation is to maximise the products of the power scattered towards Kiruna and Sodankyla, i.e. to maximise $\sin^2\psi_K \sin^2\psi_S$ where the K and S refer to Kiruna and Sodankyla, Murdin (1978). Faraday rotation is not considered.

We describe the polarisation as β anticlockwise from horizontal, see fig 3, so that in the Tromso local frame the electric vector direction $\hat{E}$ is given by

$$\hat{E} = (\cos\beta\sin\alpha + \sin\beta\sin\varepsilon\cos\alpha,\ \cos\beta\cos\alpha - \sin\beta\sin\varepsilon\sin\alpha,\ \sin\beta\cos\varepsilon)$$

where α and ε are the azimuth and elevation of the antenna. If $\hat{K}$ and $\hat{S}$ are unit vectors in the directions KQ and SQ respectively, expressed in the Tromso local frame, then

$$\cos\psi_K = \hat{E}\cdot\hat{K}$$

and

$$\cos\psi_S = \hat{E}\cdot\hat{S}$$

so $\sin^2\psi_K \sin^2\psi_S = y = |1 - (A\cos\beta + \beta\sin\beta)^2||1 - (C\cos\beta + D\sin\beta)^2|$

where

$$A = K_x\sin\alpha + K_y\cos\alpha$$
$$B = K_x\cos\alpha\sin\varepsilon - K_y\sin\alpha\sin\varepsilon + K_z\cos\varepsilon$$

and

C and D are identical with S replacing K.

Setting $\frac{dy}{d\beta} = 0$ yields after some manipulation

$$0 = T\cos2\beta + U\sin2\beta + V\cos4\beta + W\sin4\beta$$

where

$$T = (2-A^2-B^2)2CD + (2-C^2-D^2)2AB$$
$$U = (2-A^2-B^2)(D^2-C^2) + (2-C^2-D^2)(B^2-A^2)$$
$$V = (D^2-C^2)2AB + (B^2-A^2)2CD$$
$$W = (D^2-C^2)(B^2-A^2) - 4ABCD$$

which can be solved for β, to also yield ψ_K and ψ_S.

The receiver polarisation must be matched to that of the transmitter, i.e. the receiver polarisation should be in the plane of $\hat{E}$ and the scattering direction, fig 5, when $\delta = \Pi/2 - \psi$.

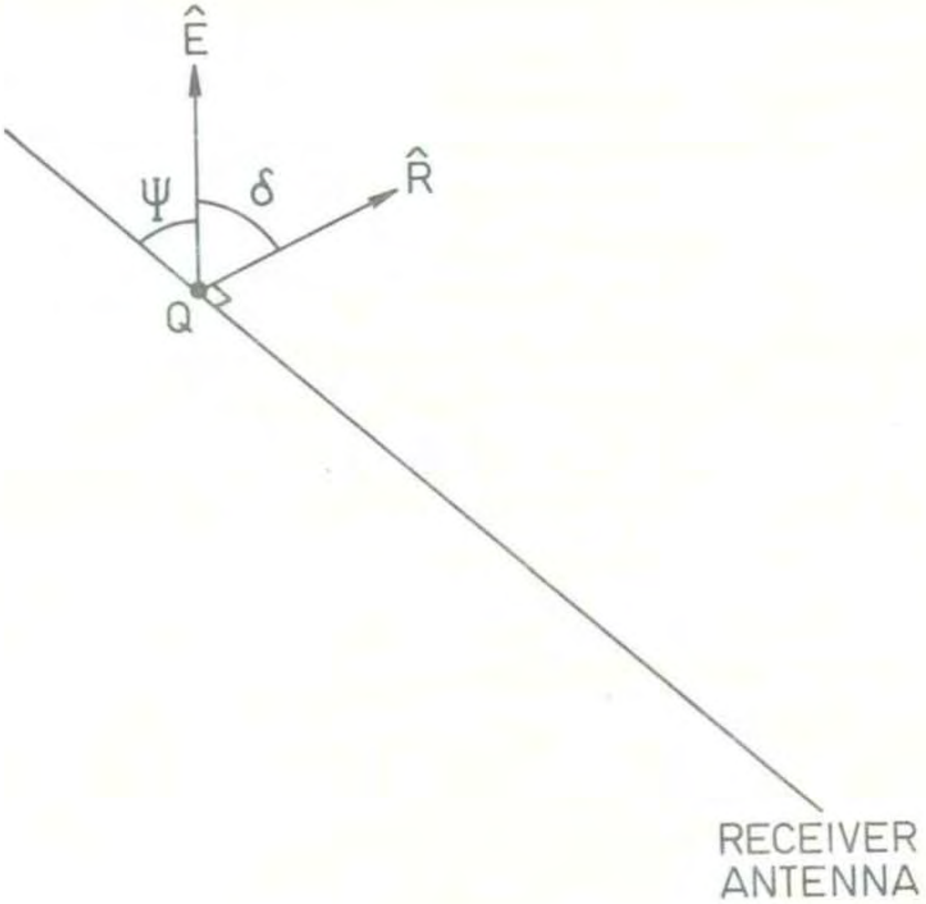

Fig 5. Receiver polarisation.

The angle between the polarisation of the receiver, $\hat{R}$, and the transmitted electric vector, $\hat{E}$, is δ and enters the expression for received power as $\cos^2\delta$, the polarisation factor. The maximum is $\cos^2\delta = \sin^2\psi$.

If $\hat{R}$ is described by β anticlockwise from horizontal in the receiver local frame and $\hat{E}$ is described in the same frame then

$$\cos^2\delta = (A\cos\beta + B\sin\beta)^2$$

where

$$A = E_x\sin\alpha + E_y\cos\alpha$$

$$B = E_x\sin\varepsilon\cos\alpha - E_y\sin\varepsilon\sin\alpha + E_z\cos\varepsilon$$

where

α and ε are the azimuth and elevation of the receiver antenna.

For an extremum in $\cos^2\delta$ we obtain

$$\tan 2\beta = \frac{2AB}{A^2-B^2} \cdot$$

Fig 6. Contours of Tromso polarisation angle that maximises the product of the received power at Kiruna and Sodankyla. Altitude 100 km. The three sites are marked as circles.

Fig. 6 shows, for an altitude of 100 km, contours of the Tromso polarisation angle β and fig. 7 the corresponding polarisation factors for Kiruna and Sodankyla, $\cos^2\delta = \sin^2\psi$. Fig. 7 is essentially the same as fig. 6.3 and fig. 6.4 in Murdin (1978).

A linearly polarised signal will suffer Faraday rotation. For the EISCAT UHF frquency the Faraday rotation is given approximately by

$$d\Omega = 10^{-18} \int N_e \cos\theta \; dh \qquad \text{radians}$$

where θ is the angle between the direction of propagation and the magnetic field, N_e is electron density and h is path length. The total electron content of the ionosphere is of the order of $10^{17}m^{-2}$, Liszka (1967), so that scattering from the ionosphere should produce a Faraday rotation of no more than 10°.

Keeping the polarisation settings at Kiruna and Sodankyla fixed but changing the Tromso polarisation angle is equivalent to introducing Faraday rotation. The polarisation factors for Kiruna and Sodankyla, $\cos^2\delta_K$ and $\cos^2\delta_S$, are plotted in fig. 8 when the Tromso polarisation angle is reduced by 10°, corresponding to a total Faraday rotation over the up and down path of 10°. This shows an improvement for Sodankyla and a deterioration for Kiruna. However the change is not large enough to indicate that Faraday rotation should modify the choice of optimum polarisation described above.

A circularly polarised signal suffers no Faraday rotation. The polarisation factor for circular polarisation, $1-0.5\sin^2\chi$ where the scattering angle is $\pi-\chi$, is plotted in fig. 9. This is inferior to the situation of fig. 8. At higher altitudes the polarisation factors for circular polarisation improve and circular polarisation may be preferable.

Fig 7. Polarisation factors, $\sin^2\psi_k$ for Kiruna (top) and $\sin^2\psi_s$ for Sodankyla (bottom) corresponding to the transmitter polarisation shown in fig 6. The polarisation factor is the fraction of power received due to polarisation mismatch between transmitter and receiver.

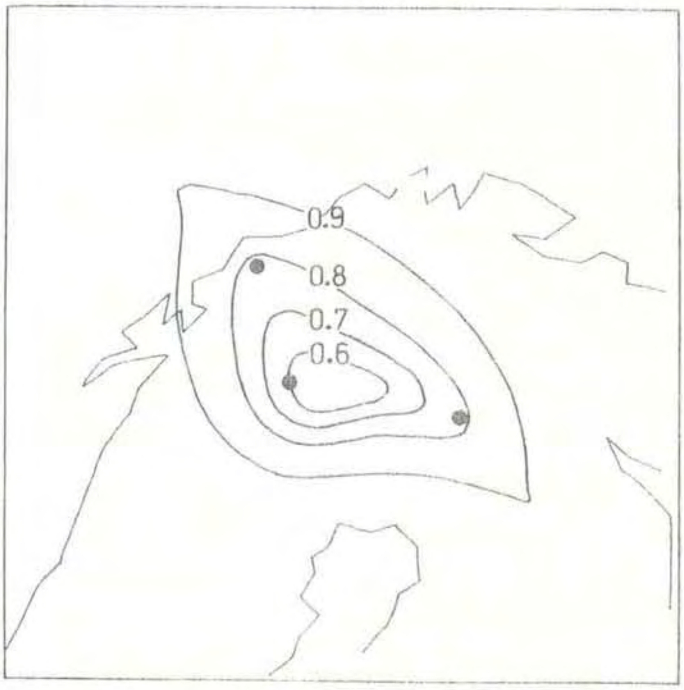

Fig 8. Polarisation factors, $\cos^2\delta$, for Kiruna (top) and Sodankyla (bottom) for the transmitter polarisation of fig 6. -10°, corresponding to total Faraday rotation of 10°.

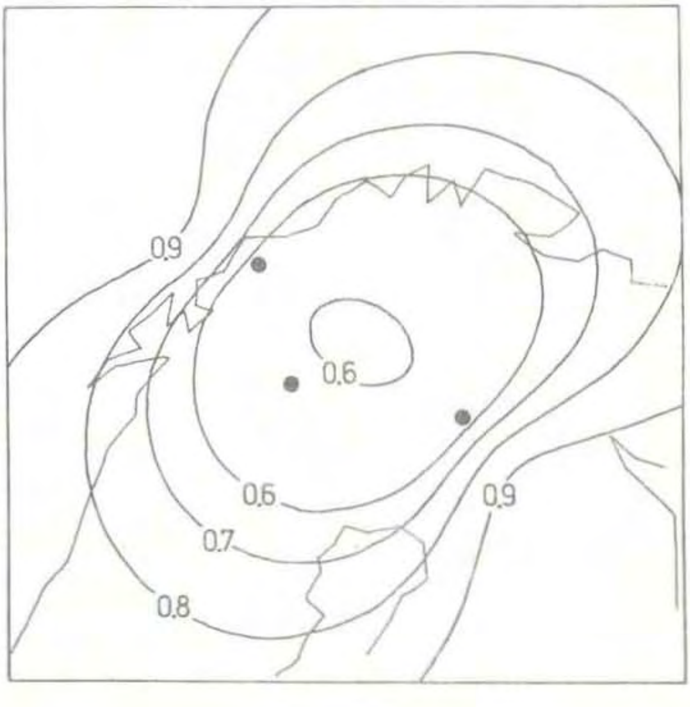

Fig 9. Polarisation factors, $1-0.5\sin^2\chi$, for Kiruna (top) and Sodankyla (bottom). Circular polarisation. Altitude 100 km.

6. Programmes

1. Subroutine COOR

Purpose: calculation of the antenna pointing directions and the mirror directions.

Input: coordinates of the scattering point Q. Either geographic latitude and longitude and altitude or range, elevation and azimuth from Tromso.

Output: in labelled common block SCAT

1. Range, azimuth and elevation of Q from Tromso, Kiruna and Sodankyla.
2. The mirror directions in geocentric coordinates, the unit vectors TQ, AQ and BQ.
3. The antenna directions in the Tromso local frame, the unit vectors TQT, KQT and SQT.
4. The angles between the transmitter and receiver antenna axes x_K and x_S.

Other routines required:

ROTATE
VECANG

2. Subroutine WIND

Purpose: constructs the matrix, MAT, that transforms the three measured components of the plasma drift velocity into the velocity vector in a geomagnetic frame.

$$\begin{bmatrix} V_{\perp E} \\ V_{\perp N} \\ V_{\parallel} \end{bmatrix} = \mathrm{MAT} \begin{bmatrix} V_T \\ V_K \\ V_S \end{bmatrix}$$

Input: latitude, longitude and altitude of scattering point Q and the mirror directions in geocentric coordinates, the unit vectors TQ, AQ and BQ.

Output: the 3 x 3 transformation matrix MAT

Other routines required:

ROTATE
MINV - matrix inversion
FELDG - IGRF 65 geomagnetic field model.

3. Subroutine POL

Purpose: to calculate the polarisation factors for Kiruna and Sodankyla.

Input: unit vectors KQT and SQT in the Tromso local frame and the azimuth and elevation of Q, the scattering point, from Tromso, Kiruna and Sodankyla.

Output: the polarisation angles (anticlockwise from horizontal) for Tromso, Kiruna and Sodankyla, POLT, POLK, POLS.
The angles between the electric vector in the transmitted wave and the receiver antenna axes, ψ_K and ψ_S.
The polarisation factors for Kiruna and Sodankyla, FACK and FACS. FAC $\leq \sin^2\psi$.

Other routines required:

ROTATE
VECANG

Comment: the code variable IND has the meaning

IND = 1 POLT, POLK and POLS input
IND = 2 $\psi_K = \pi/2$; maximise power at Kiruna
IND = 3 $\psi_S = \pi/2$; maximise power at Sodankyla
IND = 4 maximise porduct of power received at Kiruna and Sodankyla.

4. Subroutine VOLFAC

Purpose: to calculate the received signal power for a given scattering point and pulselength, Murdin (1978), for matched polarisation.

Input: range of scattering point Q from the transmitter and receiver antennas, the angle between the two antenna axes and the pulselength.

Output: $\frac{\text{signal power}}{N_e}$

5. Subroutine ROTATE

Purpose: transformation of a vector into another frame of reference

Input: 3 x 3 transformation matrix ROT. Input vector VECIN.

Output: VECOUT, the vector in the new reference frame.

6. Subroutine VECANG

Purpose: to compute the angle between two unit vectors.

7. References

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Westerlund, S., Eiscat Newsletter No. 2, March 1977.

```
      SUBROUTINE COOR(LA,LON,ALT,RANT,EL,AZ,ID)
C***********************************************************************
C            J.MURDIN    KGI                                           *
C***********************************************************************

C***********************************************************************
C      CALCULATION OF ANTENNA POINTING & SCATTERING MIRROR DIRECTIONS   *
C***********************************************************************
C***********************************************************************
C    INPUT:COORDINATES OF SCATTERING POINT Q                            *
C          ID=0   GEOGRAPHIC LATITUDE & LONGITUDE AND ALTITUDE          *
C                 LA,LON,ALT        DEGREES,KM                          *
C          ID=1   RANGE ELEVATION & AZIMUTH FROM TROMSO                 *
C                 RANT,EL,AZ        KM,DEGREES                          *
C   OUTPUT:RANGE,AZIMUTH & ELEVATION OF Q FROM T,K & S                  *
C          RANGET,AZT,ELT,RANGEK,AZK,ELK,RANGES,AZS,ELS     KM,RADIAN   *
C          ANTENNA DIRECTIONS IN TROMSO LOCAL FRAME      TQT,KQT,SQT    *
C          ANGLE BETWEEN TRANS AND RECEIVE ANTENNA AXIS   CHIK,CHIS RAD *
C          MIRROR DIRECTIONS IN GEOCENTRIC COORDS   TQ,AQ,BQ            *
C***********************************************************************

      REAL LAT,LONG,LA,LON,KQ,KQT,KQK
      DIMENSION RT(3,3),RK(3,3),RS(3,3),RTINV(3,3),
     #OT(3),OK(3),OS(3),OQ(3),TQ(3),KQ(3),SQ(3),AQ(3),BQ(3),
     #TQT(3),KQT(3),SQT(3),KQK(3),SQS(3)


      COMMON /SCAT/RANGET,AZT,ELT,RANGEK,AZK,ELK,RANGES,AZS,ELS,
     *TQT,KQT,SQT,CHIK,CHIS,TQ,AQ,BQ

      DATA A,G,F/6378.16,1.006739662,3.352871863E-3/,CON/57.29577951/

      DATA RT/0.884995448,-0.329031467,0.329425790,
     *0.308361224,0.944318958,0.114782670,
     *-0.348850129,0.0,0.937178525/,
     *RK/0.867965078,-0.349226309,0.353097167,
     *0.323483260,0.937038412,0.131596335,
     *-0.376822511,0.0,0.926285483/,
     *RS/0.824933314,-0.448539214,0.343944184,
     *0.413996637,0.893763153,0.172609996,
     *-0.384826990,0.0,0.922988726/,
     *RTINV/0.884995448,0.308361224,-0.348850129,
     *-0.329031467,0.944318958,0.0,
     *0.329425790,0.114782670,0.937178525/
      DATA OT/2107.344811,734.2675353,5955.019246/,
     *OK/2258.751708,841.8177000,5885.754287/,
     *OS/2200.075405,1104.118121,5864.471710/

        IF (ID.NE.1) THEN
          LAT=LA/CON;LONG=LON/CON;CLAT=COS(LAT);SLAT=SIN(LAT);
          T=SLAT/CLAT;CLONG=COS(LONG);SLONG=SIN(LONG);
          OQ(1)=(A/SQRT(1.+T*T/G)+ALT*CLAT)*CLONG
          OQ(2)=(A/SQRT(1.+T*T/G)+ALT*CLAT)*SLONG
          OQ(3)=A/SQRT(G+(G/T)**2)+ALT*SLAT
          DO FOR I=1,3
            TQ(I)=OQ(I)-OT(I);KQ(I)=OQ(I)-OK(I);SQ(I)=OQ(I)-OS(I);
          ENDDO
        ELSE
         ELA=EL/CON;AZA=AZ/CON;TQT(1)=-COS(ELA)*COS(AZA);
         TQT(2)=COS(ELA)*SIN(AZA);TQT(3)=SIN(ELA);
            CALL ROTATE(RTINV,TQT,TQ,AZDUM,ELDUM)
            DO FOR I=1,3
              TQ(I)=TQ(I)*RANT
              KQ(I)=-OK(I)+OT(I)+TQ(I)
              SQ(I)=-OS(I)+OT(I)+TQ(I)
              OQ(I)=OT(I)+TQ(I)
            ENDDO
```

```
    GEOCLA=ATAN2(OQ(3),SQRT(OQ(1)**2+OQ(2)**2))
    LONG=ATAN2(OQ(2),OQ(1))
    RHO=SQRT(OQ(1)**2+OQ(2)**2+OQ(3)**2)/A
    LAT=GEOCLA+SIN(2.*GEOCLA)*F/RHO+SIN(4.*GEOCLA)*F*F*
$(1./(RHO*RHO)-0.25/RHO)
    ALT=RHO-1.+0.5*F*(1.-COS(2.*GEOCLA))+F*F*(0.25/RHO-1./16.)*
$(1.-COS(4.*GEOCLA))
    ALT=ALT*A;LA=LAT*CON;LON=LONG*CON;
  ENDIF

 RANGET=SQRT(TQ(1)**2+TQ(2)**2+TQ(3)**2)
 RANGEK=SQRT(KQ(1)**2+KQ(2)**2+KQ(3)**2)
 RANGES=SQRT(SQ(1)**2+SQ(2)**2+SQ(3)**2)
 DO FOR I=1,3
   TQ(I)=TQ(I)/RANGET;KQ(I)=KQ(I)/RANGEK;SQ(I)=SQ(I)/RANGES;
 ENDDO

 AQM=0.;BQM=0.;
 DO FOR I=1,3
   AQ(I)=TQ(I)*RANGEK+KQ(I)*RANGET;AQM=AQM+AQ(I)*AQ(I);
   BQ(I)=TQ(I)*RANGES+SQ(I)*RANGET;BQM=BQM+BQ(I)*BQ(I);
 ENDDO
 AQM=SQRT(AQM);BQM=SQRT(BQM);
 DO FOR I=1,3
   AQ(I)=AQ(I)/AQM;BQ(I)=BQ(I)/BQM;
 ENDDO

 CALL ROTATE(RT,TQ,TQT,AZT,ELT)
 CALL ROTATE(RK,KQ,KQK,AZK,ELK)
 CALL ROTATE(RS,SQ,SQS,AZS,ELS)
 CALL VECANG(TQ,KQ,CHIK)
 CALL VECANG(TQ,SQ,CHIS)

 CALL ROTATE(RT,KQ,KQT,AZDUM,ELDUM)
 CALL ROTATE(RT,SQ,SQT,AZDUM,ELDUM)

 RETURN
 END
```

```
      SUBROUTINE WIND(LA,LON,ALT,TQ,AQ,BQ,MAT)
C***********************************************************************
C              J,MURDIN    KGI                                         *
C***********************************************************************

C***********************************************************************
C       VECTOR VELOCITY FROM COMPONENTS MEASURED AT THE 3 SITES        *
C       VECTOR COMPONENTS PERPENDICULAR AND ANTIPARALLEL TO B          *
C***********************************************************************
C***********************************************************************
C       INPUT:LATITUDE,LONGITUDE & ALTITUDE OF SCATTERING POINT Q      *
C             LA,LON,ALT      RADIAN,KM                                *
C             MIRROR DIRECTIONS TQ,AQ,BQ IN GEOCENTRIC COORDS          *
C      OUTPUT:MATRIX MAT                                               *
C            [VPERPE,VPERPN,VPARAL] = MAT X [VTROM,VKIRUN,VSOD ]       *
C                    [] DENOTES COLUMN VECTOR                          *
C***********************************************************************

      REAL LA,LON,LAT,LONG,RQ(3,3),
     *TQ(3),TQQ(3),AQ(3),AQQ(3),BQ(3),BQQ(3),
     *BPARAL(3),BPERPN(3),BPERPE(3),MAT1(3,3),MAT2(3,3),MAT(3,3),
     *V(3),B(3)
      INTEGER L(3),M(3)
      CON=57,29577951;LAT=LA/CON;LONG=LON/CON;

C     RQ ROTATES GEODETIC VECTORS INTO THE LOCAL FRAME AT P AND Q
      SLAT=SIN(LAT);CLAT=COS(LAT);SLON=SIN(LONG);CLON=COS(LONG);
      RQ(1,1)=SLAT*CLON;RQ(1,2)=SLAT*SLON;RQ(1,3)=-CLAT;
      RQ(2,1)=-SLON;RQ(2,2)=CLON;RQ(2,3)=0.;
      RQ(3,1)=CLAT*CLON;RQ(3,2)=CLAT*SLON;RQ(3,3)=SLAT;

C     TRANSFORM SCATTERING DIRECTIONS INTO LOCAL FRAME AT P AND Q

      CALL ROTATE(RQ,TQ,TQQ,AZ,EL)
      CALL ROTATE(RQ,AQ,AQQ,AZ,EL)
      CALL ROTATE(RQ,BQ,BQQ,AZ,EL)

C     CALCULATE B FIELD WITH IGRF65 MODEL
      CALL FELDG(LA,LON,ALT,BNORTH,BEAST,BDOWN,BABS,V,B,1)
      BX=BNORTH/BABS;BY=-BEAST/BABS;BZ=BDOWN/BABS;

C     CONSTRUCT UNIT VECTORS PARALLEL AND PERP TO -B

      BPARAL(1)=BX;BPARAL(2)=BY;BPARAL(3)=BZ;
      BPERPN(1)=-ABS(BZ/SQRT(BZ*BZ+BX*BX));BPERPN(2)=0.;
      BPERPN(3)=-BX*BPERPN(1)/BZ
      BPERPE(1)=0.;BPERPE(2)=ABS(BZ/SQRT(BZ*BZ+BY*BY));
      BPERPE(3)=-BY*BPERPE(2)/BZ

      DO 50 I=1,3
      MAT1(1,I)=TQQ(I);MAT1(2,I)=AQQ(I);MAT1(3,I)=BQQ(I);
      MAT2(1,I)=BPERPE(I);MAT2(2,I)=BPERPN(I);MAT2(3,I)=BPARAL(I);
   50 CONTINUE

      CALL MINV(MAT1,3,D,L,M)
      DO 60 J=1,3
      DO 60 I=1,3
      MAT(I,J)=0.
      DO 60 K=1,3
   60 MAT(I,J)=MAT(I,J)+MAT2(I,K)*MAT1(K,J)
      RETURN
      END
```

```
      SUBROUTINE POL(KQT,SQT,AZT,ELT,AZK,ELK,AZS,ELS,POLT,POLK,POLS,
     #INK,PSIK,PSIS,FACKK,FACSS)
C**********************************************************************
C             J.MURDIN     KGI                                        *
C**********************************************************************

C**********************************************************************
C       CALCULATION OF TRANSMITTER AND RECEIVER LINEAR POLAKIATION    *
C       WITH THE OPTION TO MAXIMISE THE PRODUCT OF THE RECEIVED       *
C       POWER AT KIRUNA & SODANKYLA                                   *
C**********************************************************************
C**********************************************************************
C      INPUT: UNIT VECTORS KQT AND SQT (IN TROMSO LOCAL FRAME)        *
C             AZIMUTH AND ELEVATION OF Q FROM TROMSO,KIRUNA & SODANKYLA *
C             AZT,ELT,AZK,ELK,AZS,ELS   RADIANS                       *
C     OUTPUT: POLARISATION FOR T,K,& S   POLT,POLK,POLS               *
C                      ANTI-CLOCKWISE FROM HORIZONTAL    RADIANS      *
C             ANGLE BETWEEN E VECTOR AND RECEIVE ANTENNA AXIS  PSIK,PSIS *
C             POLARISATION FACTORS FOR K & S      FACKK,FACSS         *
C      INK=1  POLT,POLK & POLS ARE INPUT TO ROUTINE                   *
C      INK=2  PSIK=PI/2   MAXIMUM POWER AT KIRUNA                     *
C      INK=3  PSIS=PI/2   MAXIMUM POWER AT SODANKYLA                  *
C      INK=4  MAXIMISE PSOD*PKIR                                      *
C             E IS UNIT VECTOR PERP TO TQ ANGLE BETA ANTICLOCKWISE    *
C             FROM HORIZONTAL IN GEOCENTRIC FRAME                     *
C             ET EK ES IS SAME VECTOR IN LOCAL FRAMES                 *
C**********************************************************************

      REAL KQT(3),SQT(3),L,M,N,ROOT(4),E(3),ET(3),EK(3),ES(3)
     #,RK(3,3),RS(3,3),RTINV(3,3)
       DATA RTINV/0.8849954479,0.3083612244,-0.3488501289,
     #-0.3290314665,0.9443189578,0.,
     #0.3294257902,0.1147826695,0.9371785249/,
     *RK/0.867965078,-0.349226309,0.353097167,
     *0.323483260,0.937038412,0.131596335,
     *-0.376822511,0.,0.926285483/,
     *RS/0.824933314,-0.448539214,0.343944184,
     *0.413996637,0.893763153,0.172609996,
     *-0.384826990,0.,0.922988726/,PI/3.14159265/

      SAZ=SIN(AZT);CAZ=COS(AZT);SEL=SIN(ELT);CEL=COS(ELT);
      GO TO (1,2,3,4),INK

    1 CBET=COS(POLT);SBET=SIN(POLT);
      ET(1)=CBET*SAZ+SBET*SEL*CAZ;ET(2)=CBET*CAZ-SBET*SEL*SAZ;
      ET(3)=SBET*CEL
       CALL ROTATE(RTINV,ET,E,ADUM,EDUM)
       CALL ROTATE(RK,E,EK,ADUM,EDUM)
       CALL ROTATE(RS,E,ES,ADUM,EDUM)
       AA=EK(1)*SIN(AZK)+EK(2)*COS(AZK)
       BB=EK(1)*SIN(ELK)*COS(AZK)-EK(2)*SIN(ELK)*SIN(AZK)+EK(3)*COS(ELK)

       IF(INK.EQ.1) THEN
         FACKK=(AA*COS(POLK)+BR*SIN(POLK))**2
       ELSE
         RET=0.5*ATAN(2.*AA*BB/(AA*AA-BB*BB));IF(AA.FQ.BB) BET=PI/4.;
         F1=(AA*COS(RET)+BB*SIN(BET))**2
         F2=(AA*COS(BET+PI/2.)+BB*SIN(BET+PI/2.))**2
            IF(F1.GT.F2) THEN
              POLK=BET;FACKK=F1;
            ELSE
              POLK=BET+PI/2.;FACKK=F2;
            ENDIF
         IF(POLK.LT.0.) POLK=POLK+PI
       ENDIF

       AA=ES(1)*SIN(AZS)+ES(2)*COS(AZS)
```

```
       BB=ES(1)*SIN(ELS)*COS(AZS)-ES(2)*SIN(ELS)*SIN(AZS)+ES(3)*COS(ELS)
       IF(INK.EQ.1) THEN
         FACSS=(AA*COS(POLS)+BB*SIN(POLS))**2
       ELSE
         BET=0.5*ATAN(2.*AA*BB/(AA*AA-BB*BB));IF(AA.EQ.BB) BET=PI/4.;
         F1=(AA*COS(BET)+BB*SIN(BET))**2
         F2=(AA*COS(BET+PI/2.)+BB*SIN(BET+PI/2.))**2
            IF(F1.GT.F2) THEN
              POLS=BET;FACSS=F1;
            ELSE
              POLS=BET+PI/2.;FACSS=F2;
            ENDIF
         IF(POLS.LT.0.) POLS=POLS+PI
       ENDIF

      CALL VECANG(KQT,ET,PSIK)
      CALL VECANG(SQT,ET,PSIS)
      RETURN

    2 POLT=ATAN((KQT(1)*SAZ+KQT(2)*CAZ)/
     *(KQT(2)*SEL*SAZ-KQT(1)*SEL*CAZ-KQT(3)*CEL))
      IF(POLT.LT.0.) POLT=POLT+PI
      GO TO 1

    3 POLT=ATAN((SQT(1)*SAZ+SQT(2)*CAZ)/
     *(SQT(2)*SEL*SAZ-SQT(1)*SEL*CAZ-SQT(3)*CEL))
      IF(POLT.LT.0.) POLT=POLT+PI
      GO TO 1

    4 A=KQT(1)*SAZ+KQT(2)*CAZ
      C=SQT(1)*SAZ+SQT(2)*CAZ
      B=KQT(1)*SEL*CAZ-KQT(2)*SEL*SAZ+KQT(3)*CEL
      D=SQT(1)*SEL*CAZ-SQT(2)*SEL*SAZ+SQT(3)*CEL

      L=2.-B*B-A*A;M=B*B-A*A;N=2.*A*B;P=2.*C*D;Q=D*D-C*C;R=2.-D*D-C*C;

      S=L*P+R*N;T=L*Q+R*M;U=M*P+Q*N;V=M*Q-P*N;

      IND=0;Y1=S+U;
      IF(ABS(Y1).GT.0.001) GO TO 40
      IND=IND+1;ROOT(IND)=0.;Y1=0.;
   40 DO 100 I=1,50
      THET=FLOAT(I)/50.*6.28318
      Y2=S*COS(THET)+T*SIN(THET)+U*COS(2.*THET)+V*SIN(2.*THET)
      IF(ABS(Y2).LT.0.001) GO TO 50
      IF(Y1/Y2.LT.0.) GO TO 60
      Y1=Y2
      GO TO 100
   50 IND=IND+1
      ROOT(IND)=THET;Y1=0.;
      Y1=0.
      GO TO 100
   60 THET1=THET-0.125664
      THET2=THET
   70 THET=(Y2*THET1-Y1*THET2)/(Y2-Y1)
      Y3=S*COS(THET)+T*SIN(THET)+U*COS(2.*THET)+V*SIN(2.*THET)
      IF(ABS(Y3).LT.0.001) GO TO 50
      IF(Y1/Y3.LT.0.0) GO TO 80
      Y1=Y3;THET1=THET;
      GO TO 70
   80 Y2=Y3
      THET2=THET
      GO TO 70
  100 CONTINUE
      FACMAX=0.
      DO 110 I=1,IND
      BETA=ROOT(I)/2.
      ETA=A*COS(BETA)+B*SIN(BETA)
      SPI=C*COS(BETA)+D*SIN(BETA)
      FAC=(1.-ETA*ETA)*(1.-SPI*SPI)
      IF(FAC.LT.FACMAX) GO TO 110
      BETAMA=BETA
      FACMAX=FAC
  110 CONTINUE
      POLT=BETAMA
      GO TO 1
      END
```

```
      SUBROUTINE VOLFAC(R1,R2,CHR,T,FAC)
C***********************************************************************
C               J.MURDIN    KGI                                        *
C***********************************************************************

C***********************************************************************
C       CALCULATION OF RECEIVED POWER FOR GIVEN SCATTERING POINT       *
C       AND PULSELENGTH        MURDIN  KGI REPT:1 1978                 *
C***********************************************************************
C***********************************************************************
C      R1 IS DISTANCE TRANSMITTER-Q AND R2 IS DISTANCE RECEIVER-Q      *
C       DISTANCES IN KM                                                *
C      CHR=ANGLE BETWEEN TQ AND RQ                                     *
C      T IS THE PULSE LENGTH IN MICROSECS                              *
C      SIGNAL(WATT) = 1.0 E-30 X NE(M-3) X FAC / (1+TE/TI)             *
C       NO POLARISATION MISMATCH                                       *
C       PEAK POWER = 2MW     ANTENNA EFFICIENCY 0.71 TROMSO   ELSE 0.66 *
C***********************************************************************

      R12=R1*R1
      IF(CHR.EQ.0.) GO TO 5
      S=SIN(CHR)
      C=COS(CHR)

      R22=R2*R2
      Z=S*T*23.44885/((1.+C)*SQRT(R22+R12))
      ZZ=ERF(Z)
      FAC=   ZZ*6.796182E4/(S*SQRT(R12+R22))
      RETURN
    5 FAC=4.836108E5*T/R12
      RETURN
      END



      SUBROUTINE ROTATE(ROT,VECIN,VECOUT,AZIM,ELEV)
C***********************************************************************
C               J.MURDIN    KGI                                        *
C***********************************************************************

C***********************************************************************
C      VECOUT=ROT*VECIN        ALSO OUTPUT ARE THE AZIMUTH AND ELEVATION  *
C                              OF VECOUT IN THE NEW REF FRAME          *
C***********************************************************************

      DIMENSION ROT(3,3),VECIN(3),VECOUT(3)
      DO 100 I=1,3
      VECOUT(I)=0.
      DO 100 J=1,3
  100 VECOUT(I)=VECOUT(I)+ROT(I,J)*VECIN(J)
      AZIM=ATAN2(VECOUT(2),-VECOUT(1))
      IF(AZIM.LT.0.) AZIM=AZIM+6.283185308
      ELEV=ATAN2(VECOUT(3),-VECOUT(1)/COS(AZIM))
      RETURN
      END



      SUBROUTINE VECANG(VEC1,VEC2,ANGLE)
C***********************************************************************
C               J.MURDIN    KGI                                        *
C***********************************************************************

C***********************************************************************
C      ANGLE IS THE ANGLE BETWEEN UNIT VECTORS VEC1 AND VEC2           *
C***********************************************************************
      REAL VEC1(3),VEC2(3)
      CANGLE=VEC1(1)*VEC2(1)+VEC1(2)*VEC2(2)+VEC1(3)*VEC2(3)
      ANGLE=1.570796
      IF(CANGLE.EQ.0.) RETURN
      IF(ABS(CANGLE).GE.1.0) CANGLE=1.0*CANGLE/ABS(CANGLE)
      S=SQRT(1.-CANGLE*CANGLE)
      ANGLE=ATAN2(S,CANGLE)
      RETURN
      END
```

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EISCAT SCIENTIFIC ASSOCIATION
S-981 01 KIRUNA 1, SWEDEN
TELEPHONE 0980/187 40
TELEX 8754 GEOFYSK S