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THE BANDWIDTH OF A LINEAR PHASED ARRAY WITH STEPPED DELAY CORRECTIONS

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Summary.
The note discusses the bandwidth limitation which gradually developes as the beam is phased away from that beam direction which corresponds to the correct group delays. A scheme for the increase in antenna bandwidth by introducing group delay corrections in steps along the antenna aperture is then discussed and relations between the number of groups required for various bandwidths are developed.

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1.0 INTRODUCTION

The feed for a phased array cylindrical reflector VHF antenna must be some 100 m long if the necessary effective aperture is to be achieved. If the steering is done by phasing modulo 2π at the center frequency f_0 there will be some bandwidth restrictions which may only be overcome by application of delay corrections over sections of the line feed.

In what follows we shall study the bandwidth question both without and with the delay corrections applied.

2.0 POLAR DIAGRAM

We shall assume that the field distribution along the line aperture is denoted by f(x) and that it may here be assumed to be continuous. We shall, furthermore, assume that $|f(x)| = 1$ along the feed. Hence, there is no amplitude tapering involved. For a phased array f(x) will hence take the form:

$$
\begin{aligned}
f_0(x) &= e^{-2\pi i \frac{f}{c} S_0 x} & 0 < x < \frac{\lambda_0}{S_0} \\
&= e^{-2\pi i \frac{f}{c} S_0 (x - \frac{\lambda_0}{S_0})} & \frac{\lambda_0}{S_0} < x < \frac{2\lambda_0}{S_0} \\
&\vdots & \vdots \\
&= e^{-2\pi i \frac{f}{c} S_0 (x - \frac{(M-1)\lambda_0}{S_0})} & \frac{(M-1)\lambda_0}{S_0} < x < \frac{M\lambda_0}{S_0} = L
\end{aligned}
\tag{1}
$$

Here f = actual frequency

λ_0 = wavelength at center frequency f_0

$S_0 = \sin\theta_0$, where θ_0 = nominal steering angle

L = length of the array (~ 100 m)

The radiation from this aperture distribution into a direction $\theta = \mathrm{Arcsin}\, S$ is proportional to:

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$$F(S) = \int_0^L dx \cdot f(x) \cdot e^{2\pi i S x \cdot \frac{f}{c}} \qquad (2)$$

With the particular form of f(x) given above one obtains:

$$F_0(S) = \frac{\lambda_0}{S_0} e^{i\pi\frac{f}{c}(L \cdot S - \lambda_0)} \underbrace{\frac{\sin(\pi\frac{f}{c}SL)}{\sin(\pi\frac{f}{c}S\frac{\lambda_0}{S_0})}}_{I} \cdot \underbrace{\frac{\sin(\pi\frac{f}{f_0}\frac{S-S_0}{S_0})}{\pi\frac{f}{f_0}\cdot\frac{S-S_0}{S_0}}}_{II} \qquad (3)$$

The two factors which influence the power radiated are denoted by Roman numerals I and II. The former factor is a grating lobe factor, the latter is the polar diagram of a single section of the aperture over which the phase varies by 2π at the center frequency f_0, i.e. from an aperture length $\Delta x = \lambda_0/S_0$. The two factors are depicted in Figure 1.

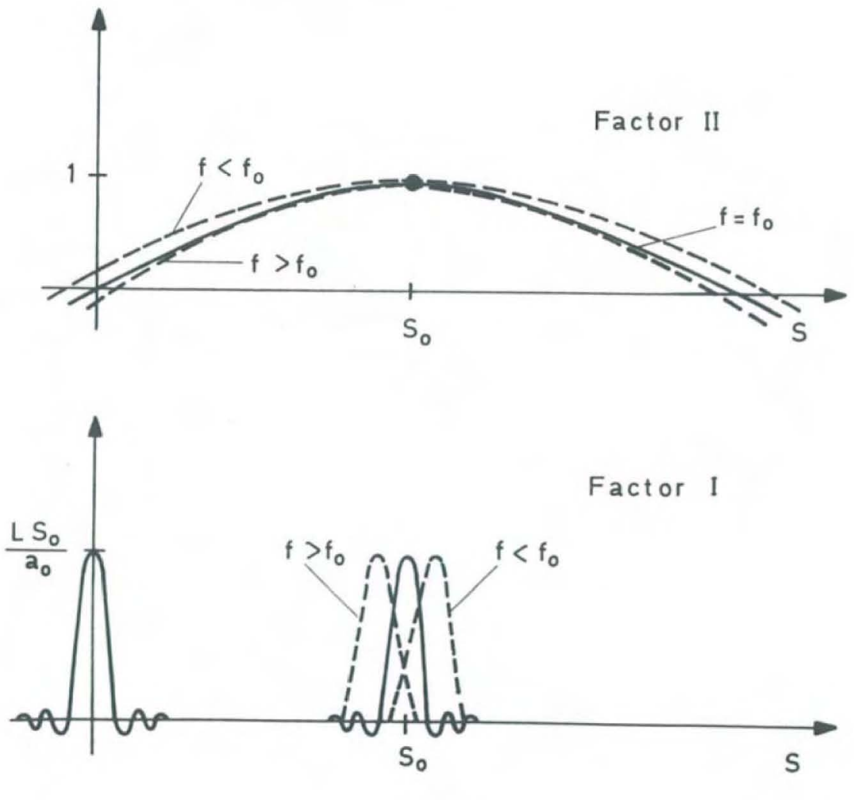

Figure 1. The two factors which determine the polar diagram $F_0(S)$.

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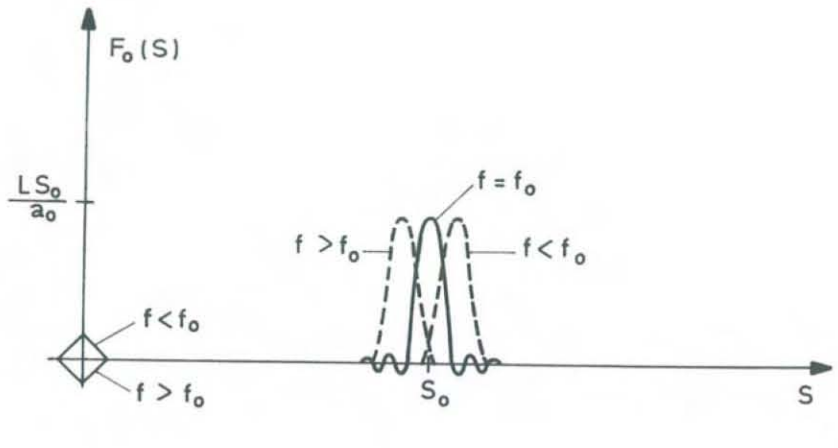

Figure 2. Resulting polar diagram.

The bandwidth is determined from the following argument: If we transmit at a frequency of f_0 and wish to receive at offset frequencies (e.g. plasma lines) we must make certain that the center of the offset frequency beams overlap the transmit frequency beam sufficiently well. A useful criterion appears to be that the center of the offset beams fall at the half-power point of the central beam:

$$\frac{\Delta f}{f_0} = - \frac{\Delta S}{S_0} \qquad (4)$$

with $\Delta S = \frac{1.4}{\pi} \cdot \frac{c}{L \cdot f_0}$ according to the above criterion. Hence we obtain for the (half-) bandwidth:

$$|\Delta f| = \frac{1.4\ c}{\pi S_0 \cdot L} \qquad (5)$$

The bandwidth is shown as function of steering angle $\theta_0 = \mathrm{Arcsin} S_0$ in Figure 3 for a feeder length of 100 m. It is apparent from this diagram that the bandwidth obtained by phasing only is quite adequate for ion-spectrum observations, but is clearly inadequate for plasma line observations. Hence, in the next section we shall consider the problem of the delay correction in order to recover the bandwidth required for plasma line observations.

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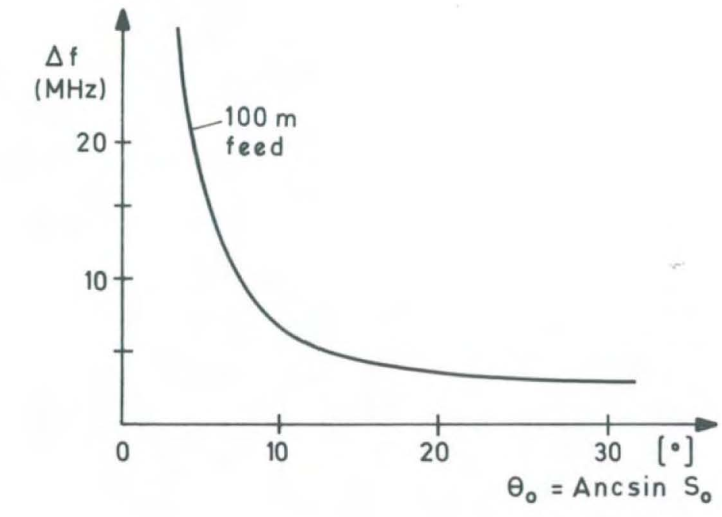

Figure 3. Half-bandwidth plotted against steering angle.

3.0 DELAY CORRECTIONS

We shall now imagine that the line feed is broken up into m equal sections and that a delay correction is applied to each section, the correction becoming progressively larger in steps along the aperture. The situation is shown for m = 4 in Figure 4.

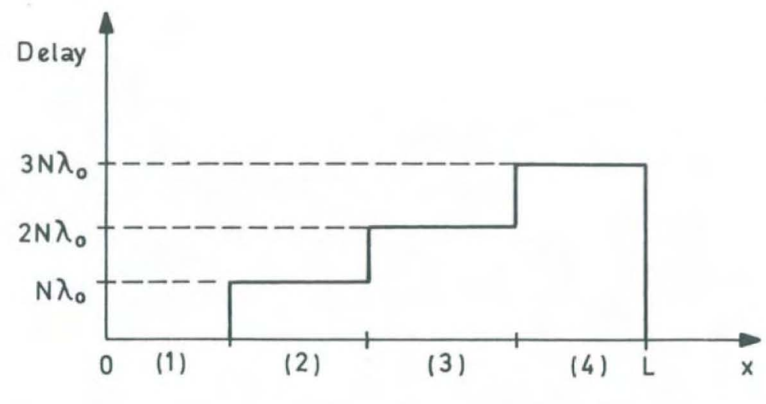

Figure 4. Delay corrections applied, m = 4.
Nominal correction: $S_1 = \frac{N \cdot m\lambda_0}{L}$.

Mathematically the delay corrections can be described by an aperture function $f_1(x)$:

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$$
\begin{aligned}
f_1(x) &= 1 && 0 < x < \frac{L}{m} \\
&= e^{-2\pi i \frac{f}{c} N \cdot \lambda_0} && \frac{L}{m} < x < \frac{2L}{m} \\
&\vdots \\
&= e^{-2\pi i \frac{f}{c}(m-1) N\lambda_0} && \frac{L(m-1)}{m} < x < L
\end{aligned}
\qquad (6)
$$

The total aperture distribution now becomes:

$$f(x) = f_1(x) f_0(x) \qquad (7)$$

If we denote the Fourier transform of $f_1(x)$ by $F_1(S)$:

$$F_1(S) = \int_0^L dx f_1(x) e^{2\pi i \frac{f}{c} Sx} \qquad (8)$$

We know that the Fourier transform of $f_1(x) f_0(x)$ must be given by the convolution of $F_1(S)$ and $F_0(S)$:

$$
\begin{aligned}
F(S) &= \int F_1(S-S') F_0(S') dS' \\
&= \int F_1(S') F_0(S-S') dS'.
\end{aligned}
\qquad (9)
$$

For $F_1(S)$ one obtains:

$$
\begin{aligned}
F_1(S) = \frac{L}{m} e^{\pi i S \frac{f}{c} \frac{L}{m}} \cdot e^{-\pi i \frac{f}{c}(N\lambda_0 - S\frac{L}{m})(m-1)} \\
\cdot \underbrace{\frac{\sin(\pi S \frac{f}{c} \frac{L}{m})}{\pi S \frac{f}{c} \frac{L}{m}}}_{I} \cdot \underbrace{\frac{\sin(\pi \frac{f}{c} m (N\lambda_0 - S\frac{L}{m}))}{\sin(\pi \frac{f}{c}(N\lambda_0 - S\frac{L}{m}))}}_{II}
\end{aligned}
\qquad (10)
$$

In order to understand the properties of $F_1(S)$ we consider first the two salient factors I and II as marked.

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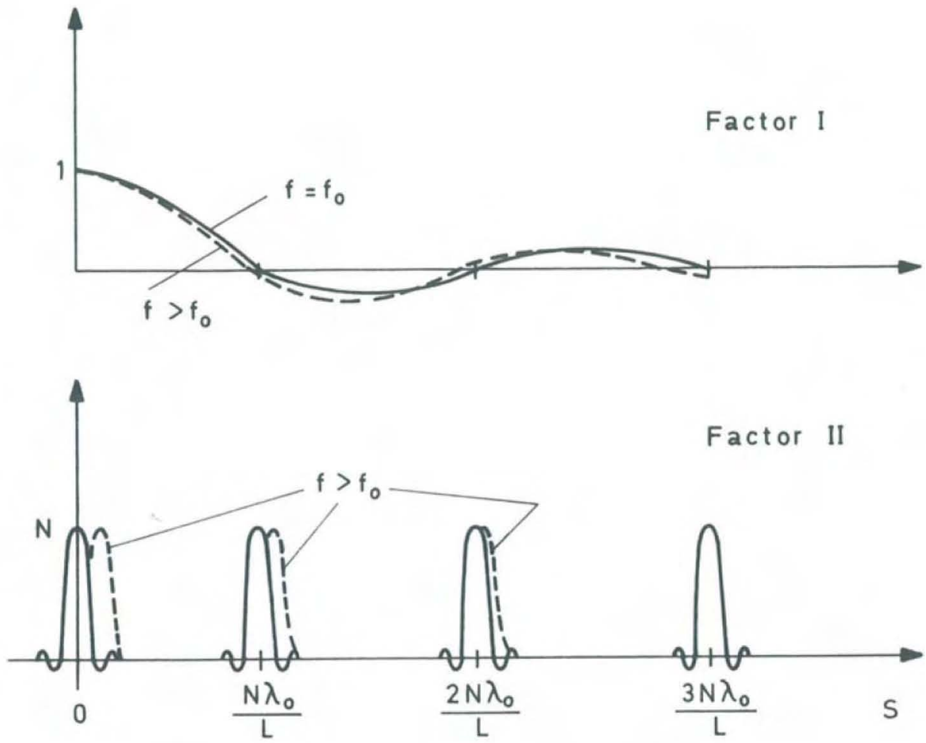

Figure 5. The two factors which jointly determine the form of $F_1(S)$.

Significant contribution will arise near S = 0 only provided Factor I remains essentially constant over the range of variation in S (near S = 0) with frequency. For the grating lobe near S = 0 we find:

$$S = \frac{N \cdot m}{L} c \left(\frac{1}{f_0} - \frac{1}{f}\right) \tag{11}$$

The variation of S with f becomes:

$$\Delta S \simeq \frac{N \cdot m \cdot \lambda_0}{L} \cdot \frac{\Delta f}{f_0} = S_1 \frac{\Delta f}{f_0} \tag{12}$$

This angular offset must be small compared with $\frac{m\lambda_0}{L}$, see Figure 5. By small we shall here mean 25%:

Hence

$$S_1 \frac{\Delta f}{f_0} \leq \frac{m \cdot \lambda_0}{4L} \tag{13}$$

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or

$$m \geq S_1 \cdot \frac{4L \cdot \Delta f}{c} \tag{14}$$

Hence, if we aim at retaining the 10 MHz bandwidth to 30°, we obtain still assuming L = 100 m:

$$m \geq 7 \tag{15}$$

One might choose m = 8 subdivisions since this allows for easier subdivision.

Let us finally turn to the convolution process. The exact calculation clearly is not easy to carry out. To carry out a calculation which is accurate enough to provide the proper insight we shall imagine that $F_0(S)$ is approximated sufficiently well by:

$$F_0(S) \approx \delta(S - S_0 \cdot \frac{f_0}{f}) \tag{16}$$

where $\delta(x)$ is a Dirac deltafunction. With this approximation we obtain for the bandwidth versus steering angle S_0:

$$|\Delta f| = \frac{1.4\ c}{\pi(S_0 - S_1) L \sqrt{1 - S_0^2}} \tag{17}$$

Note that the bandwidth is finite for $S_0 = S_1$ because of the finite bandwidth of each of the sections into which the antenna has been split in order to correct for delays.

A sketch of the bandwidth performance of the particular system assumed here is shown in Figure 6.

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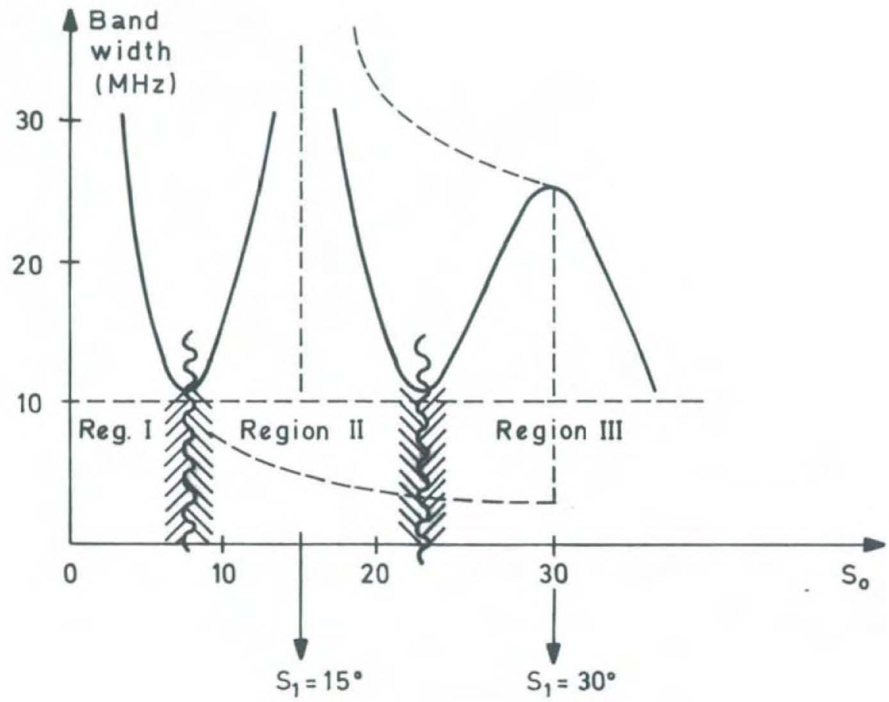

Figure 6. Sketch of achievable (half) bandwidth versus pointing "angle" S_0 when delay corrections are made at $S_0 = 15^\circ$ and at 30° for 100 m line feed. Subdivision m = 8.

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