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Abstract

After summarizing the considerations of an earlier paper on the same subject the possibility of simply using a heating frequency which is equal to the plasma frequency measured at the height of the prospective rocket apogee which will be not more than about 15^{o} from out of overhead is considered and some estimations are communicated.

In the paper on "the Modification of the Antenna Array and the Determination of the Heating Frequency for the Realization of Heating Rocket Experiments" [1] the heating frequency f_h was determined for different ionospheric layers such that the associated level of reflection vertical above the distant sub-apogee of the rocket was located at a given height h_r. According to preliminary rocket trajectory data the sub-apogee was assumed to be located at a distance of D= 76 km from the heating site and the corresponding azimuth was taken 57.8^{o} west of north.

Computations including the earth magnetic field and the earth curvature were carried out for different parabolic layers in order to determine each time the heating frequency with the above- mentioned quality and the associated direction of departure of the relevant ray, i.e. its zenith distance θ_o and azimuth angle φ_o. From these computations some preliminary information was obtained about the necessary (mean) direction of the heating beam during the rocket experiments for optimum experimental conditions. This information in turn rendered an estimate of the length of the delay cables necessary for the relevant (narrow beam) antenna array to steer the beam from out of the meridional plane into the wanted direction. (Calculations based on more recent rocket trajectory and ionospheric data will be carried out in time).

In order to obtain an additional qualitative but easily accessible survey of the problems of ray propagation associated with the heating rocket experiments, more simple calculations were performed and discussed in that paper [1] when the earth magnetic field and the earth curvature were neglected. (Such calculations can even be programmed using a small pocket computer like TI 59 if wanted).

The no earth magnetic field estimations turned out to be pessimistic in the sense that the estimated level of reflection above the subapogee of the rocket is a bit lower than that of an ordinary wave of the same frequency: it means that the value of $X = f_0^2/f^2$ at the height level of reflection is really a bit higher than estimated which implies a smaller Landau damping of the generated electron plasma waves; this again is more favourable for the experiments.

Because of the fact that ray propagation within a linear layer is comparatively very easy to handle in case when the earth magnetic field, the earth curvature and collisions are neglected, equivalent linear layer solutions have been discussed in some detail. (The equivalent linear layer has the same electron density at the height of reflection and the same electron content up to that level as the experimentally observed layer to be replaced).

With this concept rather accurate estimations for different experimental layers can be obtained of the heating frequency with the wanted quality of having its level of reflection located at a given height above the distant sub-apogee of the rocket. If the level of reflection, however, is located rather close to an ionisation maximum, the above treatment does not work too well. Unfortunately its benifit of great simplicity is further limited to the case when the earth magnetic field is neglected.

All the above-mentioned methods discussed in [1] to determine or estimate the necessary heating frequency need the relevant electron profiles up to the wanted height of reflection as an input.

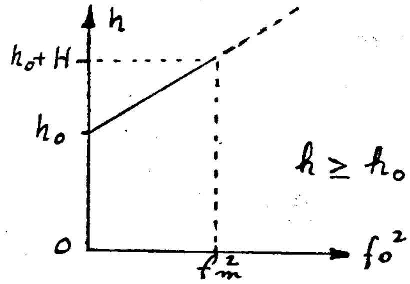
Because of the fact that the rocket apogee is not too far away from overhead a much simpler estimation may be very useful in practice. Provided that there are no horizontal gradients of importance, the heating frequency could simply be made equal to the plasma frequency measured for the height h_{apo} of the prospective apogee of the rocket. Then the height of the reflection level h_r above the distant sub-apogee is by a certain amount of kms smaller than h_{apo}. This may be favourable because then the rocket can pass the heated region twice, on its way up and down as well, provided that $\Delta h_{\perp\wedge} = h_{apo} - h_r$ is not too large because of the limited width of the heating beam. Passing the heated region twice has the benifit that there is a certain margin with regard to some dispersion not only of the rocket performance but also of the ionospheric data.

As will be shown below, this simple method appears promising especially in the realistic case when the earth magnetic field is taken into account.

It is interesting to notice again that a simple formula results for $\Delta h_{\perp\wedge}$ if for a first crude estimate the earth magnetic field etc. are neglected and a linear electron layer is assumed.

A linear layer may be described by:

$$N_e \sim f_o^2 = f_m^2 (h - h_o)/H. \qquad (1)$$



If φ_o is the angle of incidence and $f_\wedge$ is the (oblique incidence) frequency which has its reflection level at the wanted true height $h_{r\wedge}$ above the distant (D) sub-apogee, the equivalent vertical frequency is $f_\perp = f_\wedge \cdot \cos\varphi_o$. It is reflected in the same true height $h_{r\wedge}$ at vertical incidence where $f_\perp = f_o$.

From equ.(1) and with $f_o = f_\perp = f_\wedge \cos\varphi_o$ it follows:

$$(f_\wedge \cdot \cos\varphi_o / f_m)^2 = (h_{r\wedge} - h_o)/H \qquad (2)$$

or

$$h_{r\wedge} = H (f_\wedge / f_m)^2 \cos^2\varphi_o + h_o.$$

In case of vertical incidence, however, $f_\wedge$ is reflected at the different height:

$$h_{r\perp} = H\cdot(f_\wedge/f_m)^2 + h_o$$

and the wanted height difference is found to be:

$$\Delta h_{\perp\wedge} = h_{r\perp} - h_{r\wedge} = H\cdot(f_\wedge/f_m)^2 \sin^2\varphi_o. \qquad (3)$$

If $(f_\wedge/f_m)^2$ of equ.(2) and if the expression for tg φ_o, see e.g.[1]:

$$\mathrm{tg}\,\varphi_o = D/(2h_{r\wedge} - h_o)$$

as found for any linear layer are introduced into equ.(3) one finally obtains:

$$\Delta h_{\perp\wedge} = (h_{r\wedge} - h_o)\cdot D^2/(2h_{r\wedge} - h_o)^2, \quad (h_o \leq h_r). \quad (4)$$

Thus, if the heating frequency is made equal to the plasma frequency at the height $h_{r\perp} = h_{r\wedge} + \Delta h_{\perp\wedge}$ the reflection level at the distance D is located at $h_{r\wedge}$ as wanted.

If, on the other side, the heating frequency is chosen equal to the plasma frequency at the height $h_{r\wedge}$, the height of the reflection level at the distance D is by about $\Delta h_{\perp\wedge}$ smaller than the height $h_{r\wedge}$ which was aimed at.

Equ.(4) shows that in case of the above simple, no earth magnetic field approximation and with a linear layer, $\Delta h_{\perp\wedge}$ is independent of the scale height H. Further $\Delta h_{\perp\wedge}$ increases with the square of the distance D to the sub-apogee.

In Fig. 1 the height of reflection $h_{r\perp}(h_{r\wedge}) = h_{r\wedge} + \Delta h_{\perp\wedge}$ at vertical incidence is shown as a function of the height $h_{r\wedge}$ of the reflection level at the distance D= 76 km from the transmitter. h_o being the parameter, the transmitting frequency is $f = f_o(h_{r\perp})$. In case of $D^2 \ll (2h_{r\wedge} - h_o)^2$ which corresponds to the conditions of the rocket experiments the individual $h_{r\perp}(h_{r\wedge})$- curves are nearly linear.

Using Fig.1 one can either read it as $h_{r\wedge}(h_{r\perp})$ and start with a given height $h_{r\perp}$ of the vertical incidence reflection to obtain the corresponding lower height $h_{r\wedge}$ at D; or the reverse procedure $h_{r\perp}(h_{r\wedge})$ can be followed, in which case for a given $h_{r\wedge}$ the associated $h_{r\perp}$ and with this the transmitting frequency $f = f_o(h_{r\perp})$ is found for the given linear electron profile. It is a property of Fig.1 that, independent of the chosen height, $h_{r\perp}$ or $h_{r\wedge}$, which is set to a constant value (say 275 km) the missing second height ($h_{r\wedge}$ or $h_{r\perp}$) is always such that the corresponding height differences $\Delta h_{\perp\wedge} = h_{r\perp} - h_{r\wedge}$ are very nearly the same.

If one again takes D= 76 km and considers a constant height $h_{r\wedge}$ as well, $\Delta h_{\perp\wedge}$ can be plotted as a function of only h_o, see the dotted line of Fig.2. Unfortunately there does not exist such a simple relation for $\Delta h_{\perp\wedge}$ in case of other but linear layers. Therefore the corresponding curves for parabolic layers (with different heights of the layer maxima, h_{max}, as drawn in Fig.2) have been obtained in another way: at first the relative frequency f/f_c having its level of reflection at $h_{r\wedge}$ above the sub-apogee at D= 76 km was computed with the aid of a homing-in programme. Then, using the condition for vertical incidence reflections, $f = f_o$, $h_{r\perp}$ was computed for the same parabolic profile to obtain:

$$h_{r\perp} = (1 - \sqrt{1 - f^2/f_c^2}) \cdot Y_m + h_o,$$

or

$$h_{r\perp} = h_{max} - \sqrt{1 - f^2/f_c^2} \cdot (h_{max} - h_o),$$

and the wanted height difference in case of Fig.2 is:
$\Delta h_{\perp\wedge} = h_{r\perp} - 280\text{km}$.

As to be seen from Fig. 2 $\Delta h_{\perp\wedge}$ is the larger, the closer the height of the reflection level, $h_{r\wedge}$, is approaching the layer maximum or the lower the height of the lower border h_o

of the relevant layer is. Further it is clear that there do no $\Delta h_{r\perp}$ values exist if the computed (oblique incidence) frequency is higher than f_c which may happen.

In order to compare earlier, more realistic computations including the earth magnetic field with the no field estimations discussed above, the $\Delta h_{\perp\wedge}$ values are presented in a bit different way by the following two figures. In these the half thickness Y_m of the relevant parabolic layer is the independent variable and the parameter is the height h_o of its lower border. D = 76 km and $h_{r\wedge}$ = 280 km have been kept constant.

Fig.3 shows the situation in case of a flat earth with no earth magnetic field. The closer the height of the layer maximum h_{max} (=h_o + Y_m) comes to the height $h_{r\wedge}$ (=280km), the larger is $\Delta h_{\perp\wedge}$; and it is the smaller, the smaller the height difference $h_{r\wedge} - h_o$ is. In each case the corresponding transmitting frequency is equal to the plasma frequency at the height $h_{r\wedge} + \Delta h_{\perp\wedge}$ of the relevant parabolic profile: $f = f_o(280\text{km} + \Delta h_{\perp\wedge})$.

With this frequency the height of the reflection level at the distance D (=76km) is exactly $h_{r\wedge}$(=280km). If, however, as mentioned before a (lower) transmitting frequency is used which is equal to the plasma frequency at the height $h_{r\wedge}$, then the level of reflection at D = 76 km is by about $\Delta h_{\perp\wedge}$ smaller than $h_{r\wedge}$ (=280km), provided $D^2 \ll (2h_r - h_o)^2$.

The corresponding curves obtained for $\Delta h_{\perp\wedge}$ in case of an ordinary wave (i.e. when the earth magnetic field is included) are drawn in Fig.4. Because of the influence of the gyrofrequency on wave propagation, parabolic layers with different critical frequencies (different f_c/f_H-ratios) have been introduced. The different kinds of lines (full, broken, dotted) belong to the different critical frequencies of the relevant layers (4, 5, 6 MHz).

As before the individual $\Delta h_{\perp\wedge}$-curves of Fig.4 tend upwards when the height of the layer maximum h_{max} (=h_o + Y_m) approaches $h_{r\wedge}$(=280 km); and each curve belonging to a larger

h_o (=to a smaller difference between $h_{r\Lambda}$ and h_o) shows up smaller $\Delta h_{\perp\Lambda}$-values compared to a corresponding curve with a smaller h_o.

The main difference between Fig.3 and Fig.4 is that the $\Delta h_{\perp\Lambda}$-values of orinary waves, Fig.4, are smaller and not so variable over the displayed Y_m- and h_o- range than the no magnetic field values of Fig.3. Even if the level of reflection at $h_{r\Lambda}$ (=280km) is rather close to the relevant layer maximum, the $\Delta h_{\perp\Lambda}$-curves of Fig.4 do only exhibit moderate increases.

On the whole do all the different $\Delta h_{\perp\Lambda}$-values of Fig.4 not deviate by more than about ± 1km from their mean (of about 3.25km) in spite of the rather different critical frequencies and the different h_o- and Y_m- values assumed. It further appears evident from the corresponding no magnetic field estimations that the variability of the $\Delta h_{\perp\Lambda}$-values with different linear or exponential layers (for exponential layers see e.g. [1]) is smaller than in case of parabolic layers because of the lack of an ionisation maximum.

One of the reasons why the $\Delta h_{\perp\Lambda}$-values in case of ordinary waves (i.e. when the earth magnetic field is taken into account) are not so variable with the different layer parameters as in case of the corresponding no earth magnetic field estimations may be seen from Fig.5: In that figure the heights of the vertical incidence reflections are displayed for waves the frequencies of which were computed such that the reflection level above the sub-apogee at D= 76km (57,8^o west of north from the transmitter) was always 280 km. The Parabolic layer had a critical frequency of 4 MHz and a height of the layer maximum of 300km; f_H = 1.4 MHz and a dip angle of 78^o were assumed corresponding to the ionosphere above Tromsø.

The curve belonging to the no magnetic field estimation is included. In the latter case the frequencies of the waves with the above mentioned qualitiy to be reflected in h_r= 280km at D = 76km attain values of more than 4 MHz for $h_c \lesssim$ 161.35 km. Consequently $\Delta h_{\perp\Lambda}$ attains its maximum possible value of h_{max}-280km = 20km for $h_o \approx$ 161.35km and the above parabolic layer is transparent for frequencies at vertical incidence which are reflected in $h_{r\wedge} = 280km$ at D= 76km when $h_o \lesssim 161.35$ km.

In case of ordinary waves and for the parameters given in Fig.5 the corresponding frequencies do increase with decreasing h_o as well. They always remain, however, a bit below 4 MHz such that $\Delta h_{\perp\wedge} = h_{r\perp} - 280km$ even displays a maximum which does not exceed about 4km in the above example.

Finally the stability of the reflection level and the values of $X_r = f_o^2/f_r^2$ at the wanted height (of $h_{r\wedge} = 280km$) above the sub-apogee (at D= 76km, 57.8° west of north from the transmitter) have been considered when the earth magnetic field was included. These calculations have been compared to the correspondig no earth magnetic field estimations which have already been presented in Fig. 14 of Rose [1].

Two parabolic layers, with $f_c = 4MHz$, $h_{max} = 400km$ (curves 1) and $f_c = 4MHz$, $h_{max} = 300km$ (curves 2) have been looked at, see Fig.6. As far as the stabilities $(1/f_r)(df_r/dh_r)_{D,h_r}$ are concerned there is no prominent difference between the accurate calculations including the B-field and the no earth magnetic field estimations except for small h_o- values when the height of reflection is rather close to the layer maximum at the same time, see the left end of curves "2" of Fig.6. In that case is the stability a bit but not very much greater when the earth magnetic field is included.

Some values of $X_r = f_o^2/f_r^2$ at the level of reflection above the sub-apogee are also indicated in Fig.6 for both the calculations. As mentioned already earlier X_r is greater when the earth magnetic field is included. The differences between the corresponding X_r-values tend to vanish, however, when h_o comes close to $h_{r\wedge} = 280km$.

As to the determination of the heating frequency the following conclusions can be drawn from the above considerations: Because of the fact that the apogee of the rocket is not too far away from overhead there generally is no great height difference between the vertical incidence level of reflection and the reflection level above the sub-apogee. Only in case when $h_{r\Lambda}$ is rather close to a layer maximum (e.g. of a parabolic layer) $\Delta h_{\perp\Lambda}$ tends to increase. This increase, however, is rather moderate for ordinary waves compared to the case of wave propagation without the earth magnetic field.

If therefore, ionospheric measurements are available from Tromsø in the near future from which some statistical information on the ionospheric layers during the prospective launch time can be deduced, the most probable value of $\Delta h_{\perp\Lambda}$ and the associated (probably small) deviations of the individual $\Delta h_{\perp\Lambda}$-values from the mean can be determined. If then the heating frequency is made equal to the plasma frequency at a certain height above the ground (which is to be computed from the expected ionospheric and final rocket trajectory data) the level of reflection and with it the heated reagion will be such that the rocket is most likely to pass it twice, on its way up and down as well, remaining at the same time within the antenna beam.

With the above concept the work to be done (e.g. by EISCAT) for the determination of the heating frequency is considerably reduced compared to the case when a complete electron profile up to the height of the rocket apogee is needed.

Of course, even now it is not only the plasma frequency at a predetermined height level which has to be monitored. At the same time some additional measurements have to assure that sufficiently stable propagation conditions prevail with no important horizontal gradients of ionisation. These measurements must obviously include some checks of the electron profile so to avoid negative gradients of ionisation in the vicinity of $h_{r\Lambda}$ or higher electron densities below.

Acknowledgements

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References

1 Rose, G.: The modification of the antenna array and the determination of the heating frequency for the realization of heating rocket experiments. Internal Report, MPAE-W-46-79-11.

For further information, please consult the reference list of the above paper.

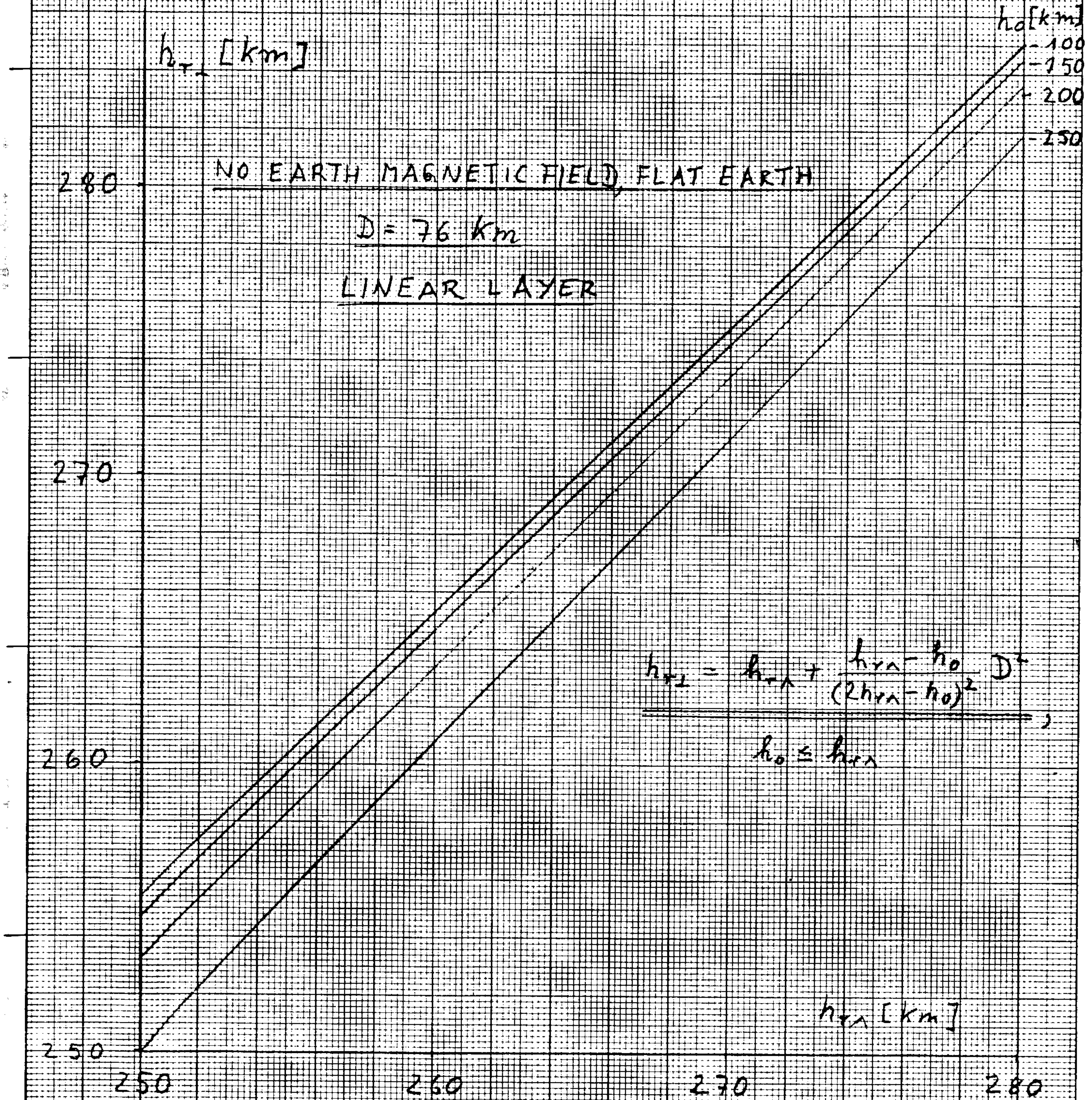


FIG. 1

THE HEIGHT OF REFLECTION OVERHEAD, $h_{r\perp}$, AS A FUNCTION OF THE HEIGHT OF THE REFLECTION LEVEL $h_{r\wedge}$ AT THE DISTANCE $D = 76$ km. $f = f_0(h_{r\perp})$.

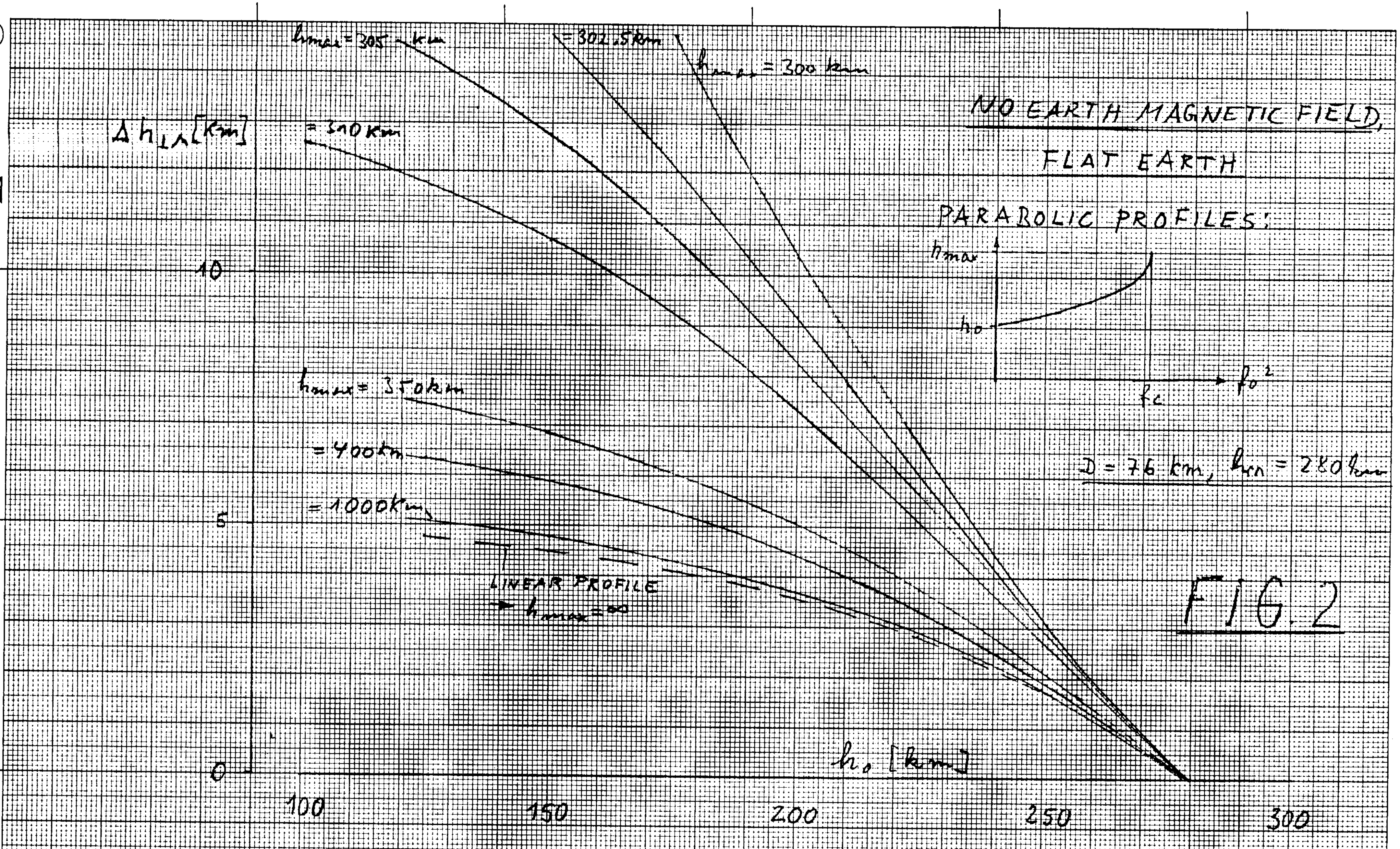


FIG. 2

THE LEVEL OF REFLECTION ABOVE THE SUB-APOGEE OF THE ROCKET IS BY $\Delta h_{\perp n}$ SMALLER THAN $h_{rn} = h_{apo}$ IF f IS CHOSEN $f = f_o(h_{apo})$ OF A HOR. STRATIFIED PARABOLIC LAYER ($D = 76$ km, $h_{apo} = 280$ km)

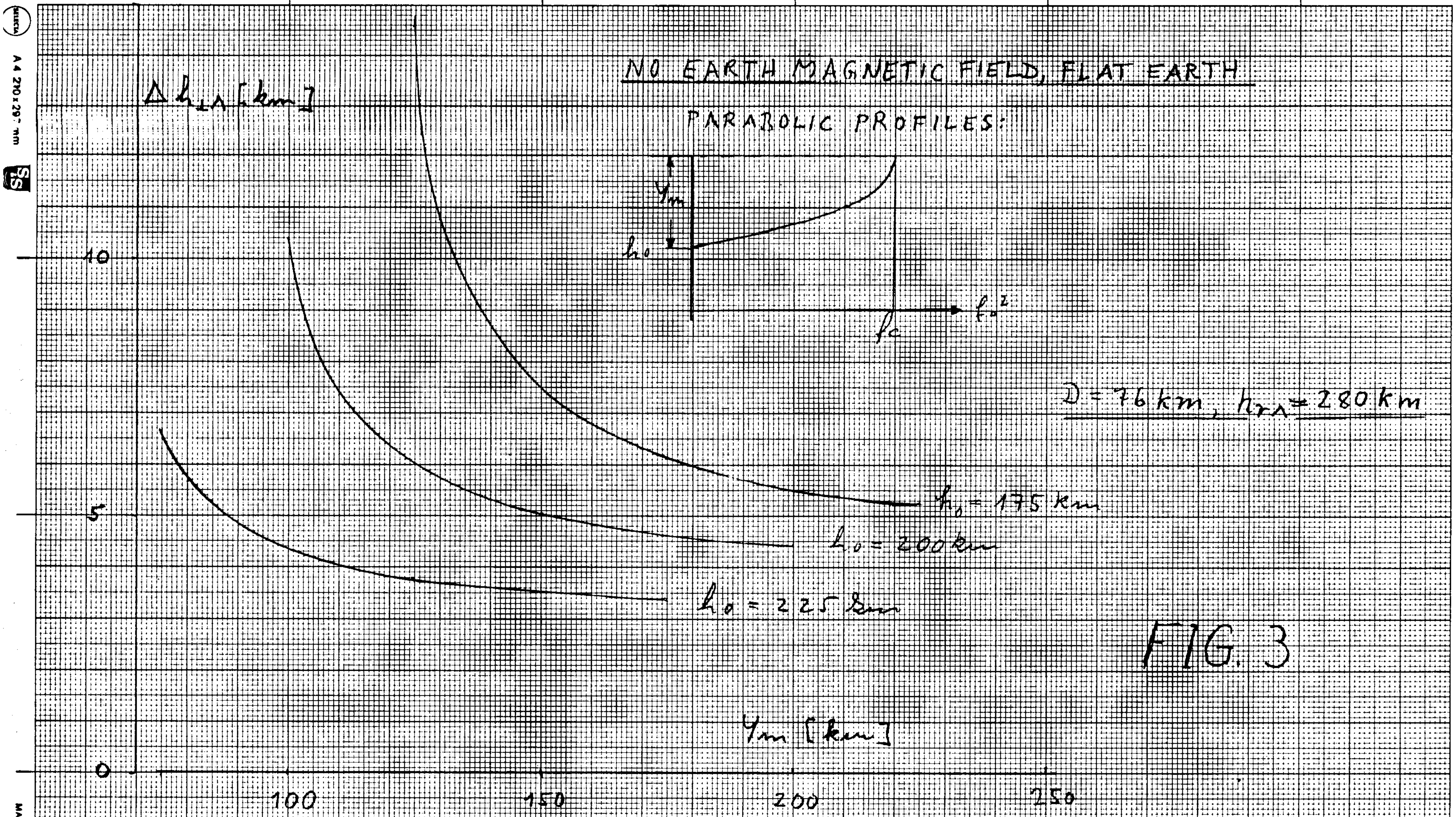


FIG. 3

THE LEVEL OF REFLECTION ABOVE THE SUB-APOGEE OF THE ROCKET IS BY ≈ Δh_{rA} SMALLER THAN $h_{rA} = h_{apo}$ IF f IS CHOSEN $f = f_0(h_{apo})$ OF THE RELEVANT HOR. STRATIFIED PARABOLIC LAYER. ($D = 76$ km, $h_{apo} = 280$ km)

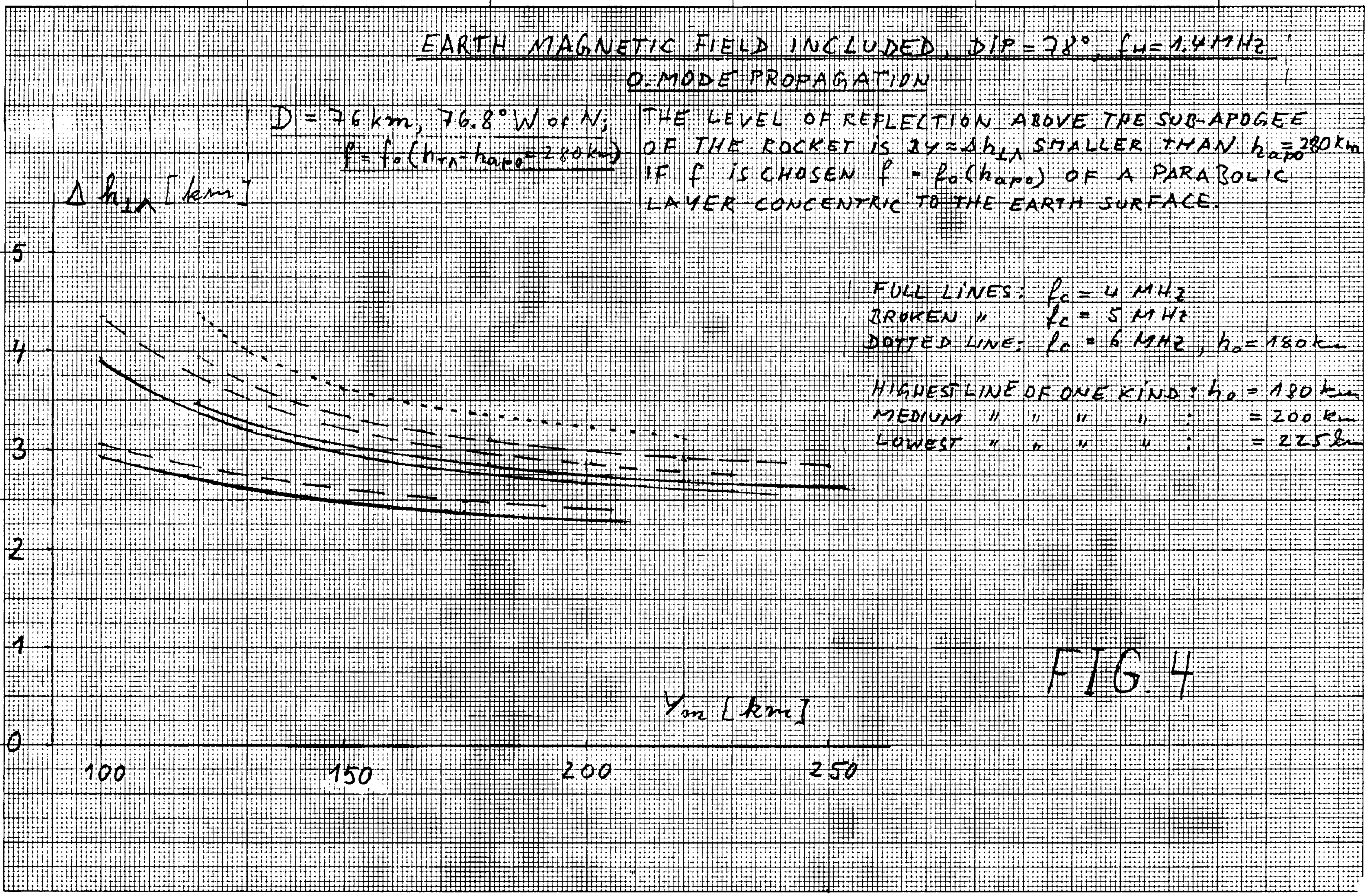

EARTH MAGNETIC FIELD INCLUDED, DIP = 78°, fH = 1.4 MHz
O. MODE PROPAGATION
D = 76 km, 76.8° W of N;
f = fo (hTA - hapo = 280 km)
THE LEVEL OF REFLECTION ABOVE THE SUB-APOGEE OF THE ROCKET IS Δy = Δh⊥A SMALLER THAN hapo = 280 km IF f IS CHOSEN f = fo(hapo) OF A PARABOLIC LAYER CONCENTRIC TO THE EARTH SURFACE.
Δh⊥A [km]
FULL LINES: fc = 4 MHz
BROKEN " fc = 5 MHz
DOTTED LINE: fc = 6 MHz, ho = 180 km
HIGHEST LINE OF ONE KIND: ho = 180 km
MEDIUM " " " " : = 200 km
LOWEST " " " " : = 225 km
5
4
3
2
1
0
100
150
200
250
Ym [km]

FIG. 4

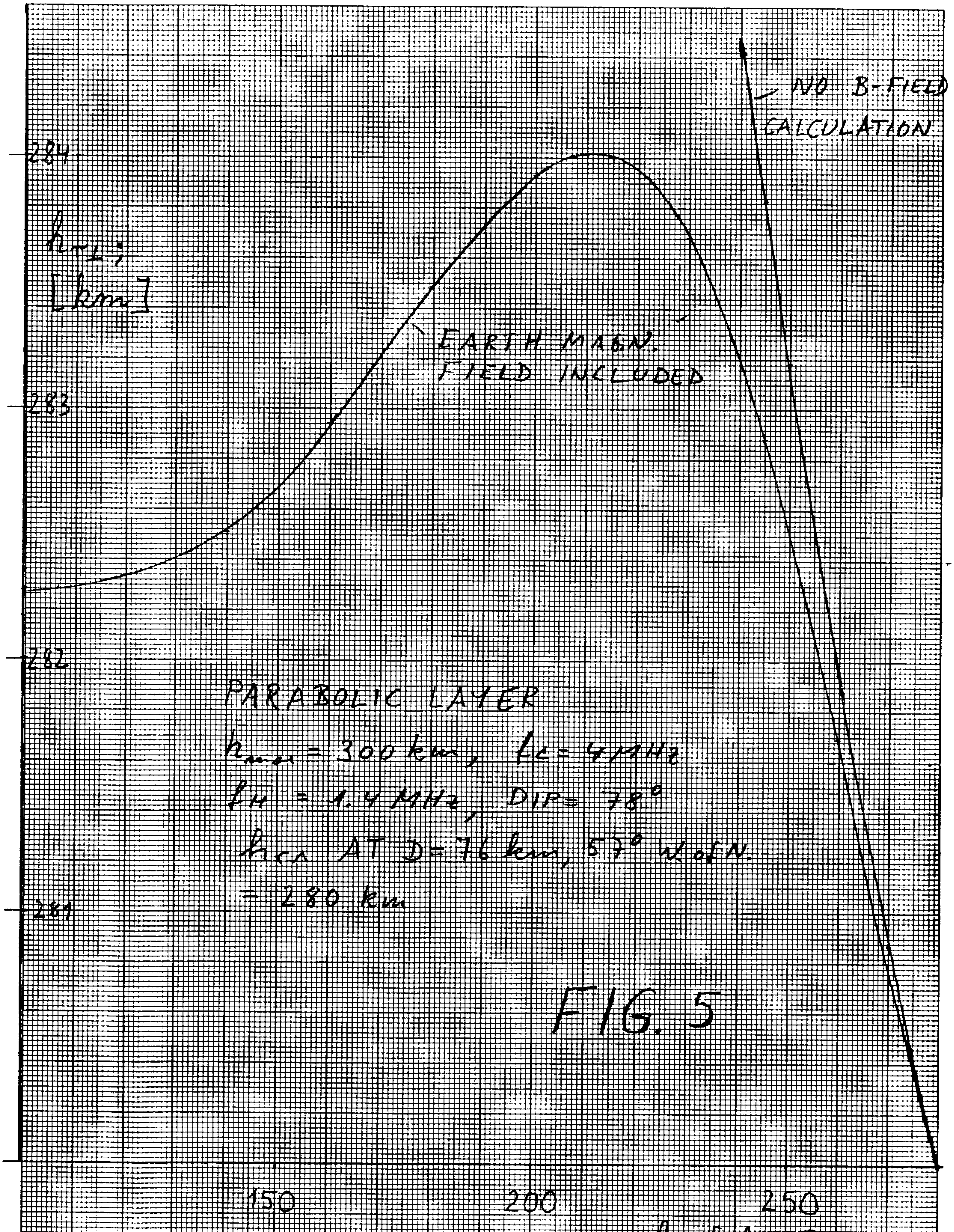
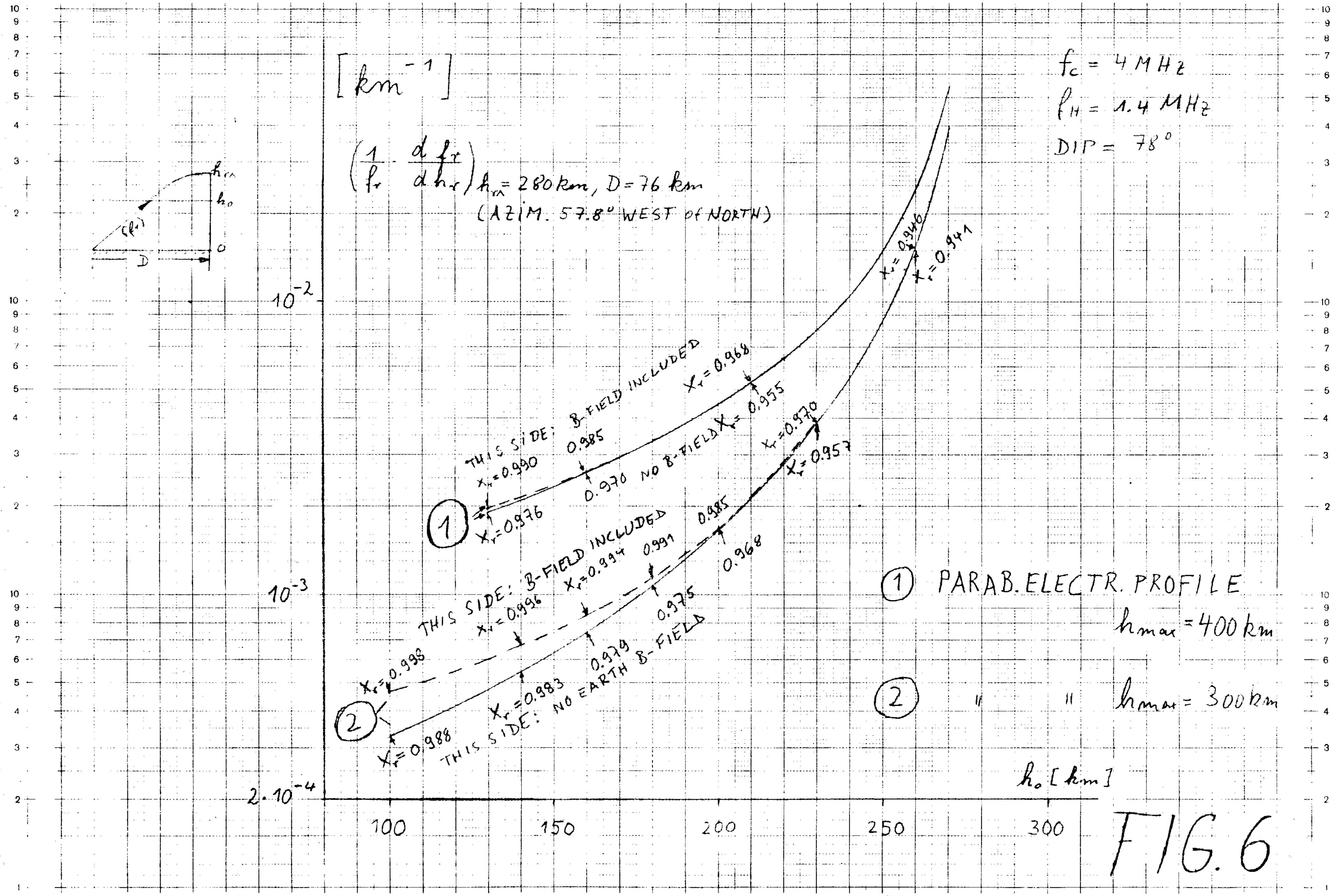


FIG. 5

HEIGHT OF THE REFLECTION LEVEL AT VERTICAL INCIDENCE WHEN THE REFL. LEVEL AT D = 76 km (57.8° W. of N.) HAS A HEIGHT OF 280 km.



[km^-1]
(1/f_r · df_r/dh_r) h_rn = 280 km, D = 76 km
(AZIM. 57.8° WEST of NORTH)
f_c = 4 MHz
f_H = 1.4 MHz
DIP = 78°
h_rn
h_o
(f_r)
0
D
10^-2
10^-3
2·10^-4
THIS SIDE: B-FIELD INCLUDED
X_r = 0.968
0.955
X_r = 0.990
0.985
0.970 NO B-FIELD
X_r = 0.976
X_r = 0.946
X_r = 0.941
X_r = 0.970
X_r = 0.957
THIS SIDE: B-FIELD INCLUDED
X_r = 0.994
0.991
0.985
0.968
X_r = 0.996
0.975
X_r = 0.998
0.979
X_r = 0.983
X_r = 0.988
THIS SIDE: NO EARTH B-FIELD
① PARAB. ELECTR. PROFILE h_max = 400 km
② " " h_max = 300 km
h_o [km]
100
150
200
250
300

FIG. 6